INDAM WORKSHOP
COMPLEX AND FOURIER ANALYSIS, AND OPERATOR THEORY 2

TITLES AND ABSTRACTS

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Evgueni Abakoumov, Universit Gustave Eiffel

Chui's conjecture in Bergman spaces

C. K. Chui conjectured in 1971 that the average gravitaional field strength in the unit disk due to unit point masses on its boundary was the smallest when these point masses were equidistributed on the circle. We will present an elementary solution to the analogous minimization problem for weighted L^2 norms, and discuss related questions concerning approximation of holomorphic functions by simple partial fractions.

This is joint work with A. Borichev and K. Fedorovskiy.

Alexandru Aleman, Lunds Universitet

Generalizations of de Branges-Rovnyak spaces

Some natural generalizations of sub-Hardy (de Branges-Rovnyak) spaces are Hilbert spaces of analytic functions in the disc, where the backward shift acts as a contraction. The sub-Bergman spaces introduced by K. Zhou are a different generalization which is interesting in its own right. These are essentially a particular case of Hilbert spaces of analytic functions in the disc, where the forward shift satises a famous hereditary inequality of S. Shimorin. The basic observation used in the talk is that such spaces are reproducing kernel Hilbert spaces whose kernel is obtained by dividing a given kernel (like the Szego or Bergman kernel) by a normalized complete Nevanlinna-Pick kernel. The aim is to deduce some general properties of these objects. We derive a useful formula for the norm and discuss some approximation results as well as some embedding theorems.

This is a report about recent joint work with F. Weistrom Dahlin as well as previous work joint with B. Malman.

Anton Baranov, St. Petersburg State University

Systems biorthogonal to exponential systems on a finite union of intervals

We study properties of a system biorthogonal to a complete and minimal system of exponentials in $L^2(E)$, where E is a finite union of intervals, and show that in the case when E is a union of two or three intervals the biorthogonal system is also complete. This generalizes a well-known result by R. Young for one interval.

The talk is based on a joint work with Yurii Belov and Alexander Kuznetsov.

Francesca Bartolucci, E.T.H. Zürich

On the connection between uniqueness from samples and stability in Gabor phase retrieval

Joint work with Rima Alaifari, Stefan Steinerberger and Matthias Wellershoff.

For every lattice Λ, we construct functions which are arbitrarily close to the Gaussian, do not agree up to global phase but have Gabor transform magnitudes agreeing on Λ. Additionally, we prove that the Gaussian can be uniquely recovered (up to global phase) in $L^2(\mathbb{R})$ from Gabor

magnitude measurements on a sufficiently fine lattice. These two facts give evidence for the existence of functions which break uniqueness from samples without affecting stability. We prove that a uniform bound on the local Lipschitz constant of the signals is not sufficient to restore uniqueness in sampled Gabor phase retrieval and more restrictive a priori knowledge of the functions is necessary. With this, we show that there is no direct connection between uniqueness from samples and stability in Gabor phase retrieval.

Alexander Borichev, Aix Marseille Université

Local behavior of the zeros of Taylor series with random and pseudo-random coefficients

For different classes of random (independent, stationary, random arithmetic sequences) and pseudo-random (exponential-polynomial, Rudin-Shapiro, Thue-Morse sequences) coefficients, we study the local zero distribution of the corresponding Taylor series.

Filippo Bracci, Universit degli studi di Roma Tor Vergata'

The (pluri)complex Poisson kernel

In this talk I will recall the different roles of the classical Poisson kernel in the unit disc (from potential theory point of view, representation formulas, dynamical point of view, rigidity results and the relations with the Julias Lemma). Then I will show how to construct the pluricomplex (i.e. maximal plurisubharmonic) Poisson kernel for strongly pseudoconvex domains and present its properties in analogy to the one-dimensional case.

Tommaso Bruno, Università degli Studi di Genova

Schrödinger operators, sub-Laplacians and their spectra on Lie groups

On a Lie group G, let $\mathcal{L}$ be a left-invariant sub-Laplacian and V be a locally integrable potential which is bounded from below. I will discuss necessary and sucient conditions for which the L^2-spectrum of the Schrö dinger operator $\mathcal{L} + V$ is purely discrete, and show how these results transfer to information on the spectrum of certain weighted sub-Laplacians on G. The talk will be mostly based on joint work with M. Calzi [1].

References

[1] T. Bruno, M. Calzi, Schrödinger operators on Lie groups with purely discrete spectrum, Adv. Math. 404, n. 108444 (2022)

Mattia Calzi, Universit degli studi di Milano

Invariant Spaces of Holomorphic Functions on Symmetric Domains

We present some old and new results on invariant (pre-)Hilbert spaces of holomorphic functions on (complex) symmetric domains. More precisely, given an irreducible bounded symmetric domain D (possibly in its unbounded realization as a Siegel domain), we consider the projective representation

$$U_s(\varphi)f = (f \circ \varphi^{-1})(J\varphi^{-1})^{s/g}$$

of the (component of the identity in the) group G of biholomorphisms of D in the space of holomorphic functions on D, where $s \in \mathbb{R}$ and g is the genus of D. For example, D may be the unit ball in $\mathbb{C}^{n+1}$ or the corresponding Siegel upper half-space $\{(\zeta, z) \in \mathbb{C}^n \times \mathbb{C} \colon \Im z - |\zeta|^2 > 0\}$, in which case $g = n + 2$.

In a series of works (cf. [4, 1, 2, 3, 5] to name only a few) the complete pre-Hilbert subspaces in which U_s induces a unitary representation of G (and satisfying some additional assumptions) have been determined and described.

In this talk we shall consider the stabilizer G_{Aff} of a point in the Šilov boundary of D in G (which corresponds to the group of affine biholomorphisms of D in a suitable unbounded realization as a Siegel domain), and determine the complete pre-Hilbert subspaces in which U_s induces a unitary representation of G_{Aff} (and satisfying some additional 'strong decency' assumption) when D has rank 1 or is of tube type.

This is joint work with M. M. Peloso.

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References

[1] Arazy, J., Fisher, S. D., The Uniqueness of the Dirichlet Space among Möbius Invariant Hilbert Spaces, *Ill. J. Math.* **29** (1985), p. 449–462.
[2] Arazy, J., Fisher, S. D., Invariant Hilbert Spaces of Analytic Functions on Bounded Symmetric Domains, in: *Operator Theory: Advances and Applications*, **48** (1990), Birkhauser Verlag, Basel, p. 67–91.
[3] Arazy, J., A Survey of Invariant Hilbert Spaces of Analytic Functions on Bounded Symmetric Domains, *Contemp. Math.* **185** (1995), p. 7–65.
[4] Vergne, M., Rossi, H., Analytic Continuation of the Holomorphic Fourier Series of a Semisimple Lie Group, *Acta Math.* **136** (1976), p. 1–59.
[5] Yan, Z., Invariant Differential Operators and Holomorphic Function Spaces, *J. Lie Theory* **10** (2000), p. 1–31.

Nikolaos Chalmoukis, Universitt des Saarlandes

Interpolation by analytic functions in Dirichlet type spaces

In this talk we shall discuss different notions of interpolation for spaces of analytic functions in the unit disc. In particular for the classical Dirichlet space we will present a characterization of simply interpolating sequences with finite associated measure in terms of capacity of some condensers. The same condition in fact solves a similar interpolation like problem on the Sobolev space $H_1(D)$. We will also present some counter examples that show how the problem of interpolation is different in the Dirichlet space compared to the Hardy space.

Elena Cordero, Universit degli studi di Torino

Symplectic Time-frequency Analysis and Applications to Quantization, Schr odinger equations and wave fronts

We introduce a symplectic time-frequency analysis. The standard time-frequency representation Short-time Fourier Transform (STFT) is replaced by the $\mathcal{A}$-Wigner distribution defined by

$$W_{\mathcal{A}}(f;g) = \mu(\mathcal{A})(f \otimes \bar{g}); \quad f;g \in L^2(\mathbb{R}^d);$$

where $\mathcal{A}$ is a $4d \times 4d$ symplectic matrix and $\mu(\mathcal{A})$ is an associate metaplectic operator. Basic examples are given by the so-called τ-Wigner distributions. Symplectic representations can be eciently employed in Quantum Mechanics and in the related study of pseudodifferential operators, paving the way for a new understanding of quantization procedures [1]. In particular, we nd subclasses of symplectic matrices $\mathcal{A}$ that can be used to dene the modulation spaces [2]. Finally, we deduce micro-local properties for pseudodierential operators in terms of the $\mathcal{A}$-Wigner wave front sets. In fact, modulation spaces and $W_{\mathcal{A}}$ representations are the frame for a new definition of wave front set, providing a sharp result for propagation of micro-singularities in the case of Schrödinger equations with quadratic Hamiltonians [3].

This is a joint project with Gianluca Giacchi (University of Bologna) and Luigi Rodino (University of Torino).

References

[1] E. Cordero and L. Rodino, Wigner analysis of operators. Part I: pseudodierential operators and wave fronts. Appl. Comput. Harmon. Anal., 58:85123, (2022).
[2] E. Cordero and L. Rodino, Characterization of modulation spaces by symplectic representations and applications to Schrödinger equations. arXiv:2204.14124v1
[3] E. Cordero, G. Giacchi and L. Rodino, Wigner Analysis of Operators. Part II: Schrödinger equations. arXiv:2208.00505.

Gian Maria DallAra, INdAM Pisa

Some recent results and open questions on the d-bar Neumann problem

I will survey aspects of the regularity theory of the d-bar Neumann problem on smooth pseudoconvex domains of complex space, highlighting the role played by transversality (or lack thereof) of Levi-null.

Karlheinz Gröchenig, University of Vienna

Sampling and complete interpolating sequences in the shift-invariant space generated by a Gaussian

We study the problem of sampling and interpolation in the shift-invariant space V^2 with Gaussian generator

$$V^2 = \Big\{ f(z) = \sum_{n\in\mathbb{Z}} c_n e^{-(z-n)^2} : \ (c_n) \in \ell^2(\mathbb{Z}) \Big\}.$$

This is a subspace of $L^2(\mathbb{R})$ that is often used as a substitute for bandlimited functions.

We give a full description of complete interpolating sequences for V^2. Roughly speaking, an increasing sequence $(\lambda_n)_{n\in\mathbb{Z}}$ is a complete interpolating sequence for V^2, if it is a perturbation $\lambda_n = n + \delta_n, n \in \mathbb{Z}$ of the integers and a local average of the deviations δ_n is smaller than $1/2$ (an Avdonin-type condition).

As a consequence, we rederive the known density conditions for sampling and interpolation in V^2.

The proof is combination of complex analysis (relation to small Fock space) and functional analytic techniques.

This is joint work with Anton Baranov and Yurii Belov, St. Petersburg State University.

Michael Hartz, Universitt des Saarlandes

Common range of co-analytic Toeplitz operators in the Drury–Arveson space

A theorem of McCarthy describes the space of all functions in the Hardy space on the unit disc belonging to the range of every co-analytic Toeplitz operator. This space is intimately related to the classical Smirnov class on the unit disc. In multivariable operator theory, the appropriate generalization of the classical Hardy space is often thought to be the Drury–Arveson space H^2_d on the unit ball in $\mathbb{C}^d$. I will talk about a generalization of McCarthy's theorem to the Drury–Arveson space. In particular, this involves understanding appropriate analogues of the Smirnov class in this setting. This is joint work with Alexandru Aleman, John McCarthy and Stefan Richter.

Alessandro Monguzzi, Università degli Studi di Bergamo

On the speed of convergence of Weyl sums on Kronecker sequences

In this talk I will report on an ongoing project with Leonardo Colzani and Bianca Gariboldi.

Given $\alpha \in \mathbb{R}^d$ and suitable weight function Φ we define the weighed discrepancy

$$\mathcal{D}_N^{\Phi,\alpha} f(x) = \Big(\sum_{n=-\infty}^{+\infty} \Phi(N^{-1}n) \Big)^{-1} \sum_{n=-\infty}^{+\infty} \Phi(N^{-1}n) f(x + n\alpha) - \int_{\mathbb{T}^d} f(y)dy$$

and we investigate the pointwise speed of convergence to zero of such discrepancy. We estimate such speed in terms of the Diophantine properties of the vectors α, properties of the weight function Φ and the smoothness of the function f.

Artur Nicolau, Universitat Autnoma de Barcelona

Linear combinations of iterates of an inner function

Convergence of linear combinations of iterates of an inner function fixing the origin will be considered. A version of the Central Limit Theorem and of the Kolmogorov- Khintchinne test will be discussed. Convergence in some classical function spaces will also be discussed.

Joaquim Ortega-Cerdà, Universitat Autònoma de Barcelona

Functionals that are maximized by reproducing kernels and applications

We show that several convex functionals are maximized by reproducing kernels in the Fock space, the Bergman space and spaces of polynomials in the sphere. This provides elementary proofs of several contractive inequalities between spaces of holomorphic functions.

Karl-Mikael Perfekt, NTNU Trondheim

Hankel operators on PaleyWiener spaces in two dimensions

I will discuss PaleyWiener spaces consisting of functions with Fourier transform supported in either a square or a disc. We focus on the description of bounded Hankel/Toeplitz operators on such spaces, or equivalently, on weak factorization. Two geometric arguments will be presented. The first relates Hankel operators in the square case to Hankel operators on the two-dimensional Hardy space $H^2(\mathbb{T}^2)$. The second shows that Neharis theorem, and thus weak factorization, does not hold in the disc-case. Based on joint papers with M. Carlsson and O. F. Brevig

José Luis Romero, Universität Wien

Zeros of the short-time Fourier transform under white noise

The short-time Fourier transform (STFT) shows how the frequency profile of a real variable function evolves with time. As it turns out, as soon as functions are impacted by even a moderate amount of noise, the zeros of their STFTs exhibit a rather rigid pattern with predictable statistics, and this information can be exploited in practice. In the important special case of Gaussian windows, STFTs of white noise can be identified with certain Gaussian entire functions. I will discuss the statistics of the zero sets of STFTs of random functions, and specially algorithms for the computation of such zeros from finite data.

The talk is based on joint work with Luis Alberto Escudero, Naomi Feldheim, Antti Haimi and Guenther Koliander.

Stefan Richter, University of Tennessee

Multivariable versions Kaluza's Lemma

Let $d \in \mathbb{N}$ and $f(z) = \sum_{\alpha \in \mathbb{N}_0^d} c_\alpha z^\alpha$ be a convergent multivariable power series in $z = (z_1, \dots, z_d)$. We present two independent conditions on the positive coefficients c_α which imply that $f(z) = \frac{1}{1-\sum_{\alpha \in \mathbb{N}_0^d} q_\alpha z^\alpha}$ for non-negative coefficients q_α. It turns out that functions of the type

$$f(z) = \int_{[0,1]^d} \frac{1}{1 - \sum_{j=1}^d t_j z_j} d\mu(t)$$

satisfy one of our conditions, whenever $d\mu(t) = d\mu_1(t_1) \times \cdots \times d\mu_d(t_d)$ is a product of probability measures μ_j on $[0, 1]$. The results have applications in the theory of Nevanlinna-Pick kernels.

This is joint work with Jesse Sautel.

Caterina Stoppato, Università degli Studi di Firenze

Variants of the Diederich-Fornaess worm domain

In 1977, Diederich and Fornaess constructed an example of smoothly bounded pseudoconvex domain $\mathcal{W} \subset \mathbb{C}^2$ with nontrivial Nebenhülle. Over the years, $\mathcal{W}$ (known as the "worm" domain amongst experts) turned out to have many other important features. For instance, it is an example of pseudoconvex domain which does not fulfill the so-called Condition R: in other words, the Bergman projection does not preserve $C^\infty(\overline{\mathcal{W}})$. For this reason, it is still unknown whether every automorphism of $\mathcal{W}$ extends to a diffeomorphism from the closure $\overline{\mathcal{W}}$ to itself. The talk will cover variants of $\mathcal{W}$ studied in collaboration with Steven Krantz, Alessandro Monguzzi and Marco Peloso.

Maria Vallarino, Politecnico di Torino

Analysis on trees with non-doubling flows

The classical Calderón-Zygmund theory was developed in the Euclidean space and, more generally, on spaces of homogeneous type, which are measure metric spaces with the doubling property. In this talk we consider innite trees endowed with locally doubling flow measures μ which are nondoubling of at least exponential growth. In this setting, we develop a Calderón-Zygmund theory and we define BMO and Hardy spaces, proving a number of desired results extending the corresponding theory as known in the classical setting. Moreover, in the particular case of the homogenous tree $\mathbb{T}_q$ endowed with a distinguished flow measure we study the boundedness properties of the first order Riesz transform associated with a Laplacian operator self-adjoint on $L^2(\mu)$.

This is a joint work with Matteo Levi, Alessio Martini, Federico Santagati and Anita Tabacco.