Combining Data Assimilation and Data-driven Sparse Sensing Placement Method For Designing Better Observation Locations for NWP

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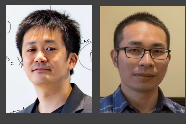


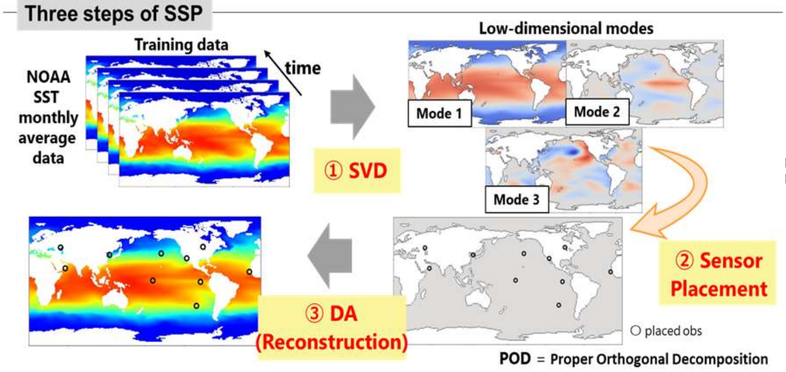
Overview of Sparse Sensing Placement (SSP)

Concept of SSP method

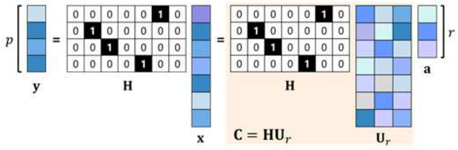
Data-driven sparse sensor placement (SSP)

→ A method for optimizing sensor locations using **sparseness behind data**
(Manohar et al. 2018)

Three steps of SSP



Mathematics



$\mathbf{x} \in \mathbb{R}^n$: state vector n : # of state
$\mathbf{y} \in \mathbb{R}^p$: observation p : # of obs
$\mathbf{a} \in \mathbb{R}^r$: amplitudes r : # of modes
$\mathbf{H} \in \mathbb{R}^{p \times r}$: obs. ope.
$\mathbf{U}_r \in \mathbb{R}^{n \times r}$: modes

amplitude of modes

Decomposition $\mathbf{x} \approx \mathbf{u}_1 \cdot a_1 + \mathbf{u}_2 \cdot a_2 + \cdots + \mathbf{u}_r \cdot a_r \Leftrightarrow \mathbf{x} \approx \mathbf{U}_r\mathbf{a}$

Reconstruction by LS $\mathbf{y}_{true} = \mathbf{H}\mathbf{x} \approx \mathbf{H}\mathbf{U}_r\mathbf{a}_{true} = \mathbf{C}\mathbf{a}_{true}$ least square regression

Determination of amplitudes

$$\Leftrightarrow \mathbf{a}_{true} \approx \begin{cases} \mathbf{C}^{\mathrm{T}}(\mathbf{C}\mathbf{C}^{\mathrm{T}})^{-1}\mathbf{y}_{true} & \text{underdetermined } (p < r) \\ (\mathbf{C}^{\mathrm{T}}\mathbf{C})^{-1}\mathbf{C}^{\mathrm{T}}\mathbf{y}_{true} & \text{overdetermined } (p \geq r) \end{cases}$$

Error covariance of estimated amplitudes

$$\langle(\mathbf{a}_{estimated} - \mathbf{a}_{true})(\mathbf{a}_{estimated} - \mathbf{a}_{true})^{\mathrm{T}}\rangle \propto \mathbf{F}^{-1} = \begin{cases} (\mathbf{C}\mathbf{C}^{\mathrm{T}})^{-1} & (p < r) \\ (\mathbf{C}^{\mathrm{T}}\mathbf{C})^{-1} & (p \geq r) \end{cases}$$

F: Fisher's Information mtx

We can estimate "x" when amplitudes "a" is determined.
➔ SSP finds obs placements to estimate amplitudes "a" accurately.

Proof (a case for overdetermined problem i.e., $p \geq r$) Manohar et al. (2018), Saito et al. (2021)

Suppose obs have Gaussian noise $\boldsymbol{\varepsilon} = N(\mathbf{0}, \sigma^2\mathbf{I})$, and $\mathbf{C}$ is trustable

$$\mathbf{a}_{estimated} = (\mathbf{C}^{\mathrm{T}}\mathbf{C})^{-1}\mathbf{C}^{\mathrm{T}}(\mathbf{y}_{true} + \boldsymbol{\varepsilon})$$

$$\begin{aligned} \langle(\mathbf{a}_{true} - \mathbf{a}_{estimated})(\mathbf{a}_{true} - \mathbf{a}_{estimated})^{\mathrm{T}}\rangle &= \langle(\mathbf{C}^{\mathrm{T}}\mathbf{C})^{-1}\mathbf{C}^{\mathrm{T}}\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^{\mathrm{T}}\mathbf{C}(\mathbf{C}^{\mathrm{T}}\mathbf{C})^{-1}\rangle \\ &= \sigma^2\langle(\mathbf{C}^{\mathrm{T}}\mathbf{C})^{-1}\mathbf{C}^{\mathrm{T}}\mathbf{C}(\mathbf{C}^{\mathrm{T}}\mathbf{C})^{-1}\rangle \\ &= \sigma^2(\mathbf{C}^{\mathrm{T}}\mathbf{C})^{-1} \\ &\propto \boxed{(\mathbf{C}^{\mathrm{T}}\mathbf{C})^{-1}} \end{aligned}$$

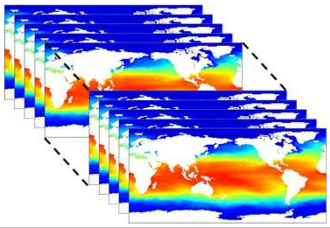
$\langle\cdot\rangle$: expected value

$\langle\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^{\mathrm{T}}\rangle = \sigma^2\mathbf{I}$

Proof of underdetermined system is little bit more complicated

Maximizing $(\mathbf{C}\mathbf{C}^{\mathrm{T}})$ or $(\mathbf{C}^{\mathrm{T}}\mathbf{C})$ minimizes $|\mathbf{a}_{true} - \mathbf{a}_{estimated}|$
➔ good estimate on **x**.

Preliminary Experiments w/ SST

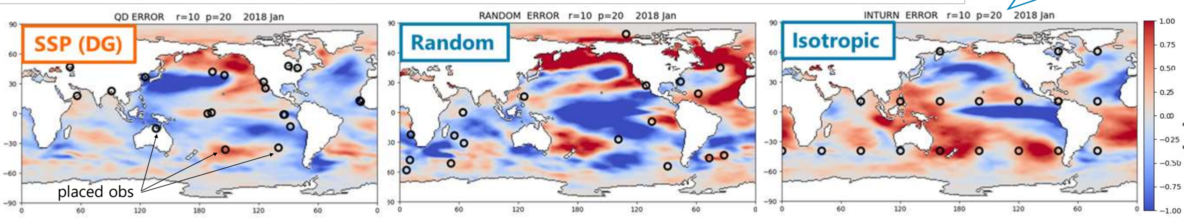


Experiments

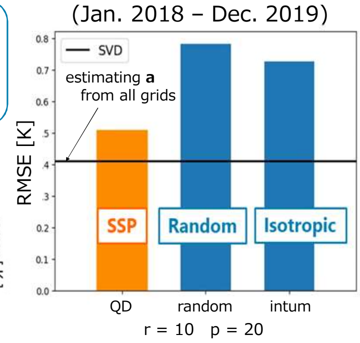
- Grid points (=candidates of sensor place) : n=43799
- Training Period : 30 years (Jan. 1988 ～ Dec. 2017)
- Test Period : 2 years (Jan. 2018 ～ Dec. 2019)
- Data : monthly average sea surface temperature

Sparce Sensor Placement

Estimation Error $(\mathbf{x}_{estimation} - \mathbf{x}_{true})$
# of mode $r = 10$
# of sensors $p = 20$

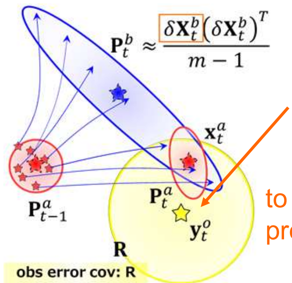


Global-mean RMSE
(Jan. 2018 – Dec. 2019)



An application for NWP (Experiments w/ SPEEDY)

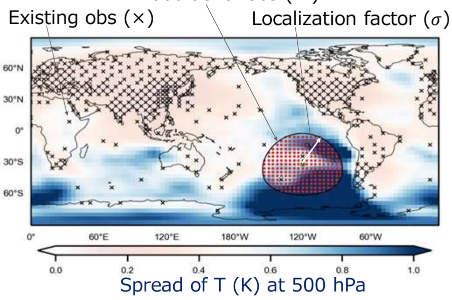
Experiments with Ensemble Kalman Filter



SVD & SSP

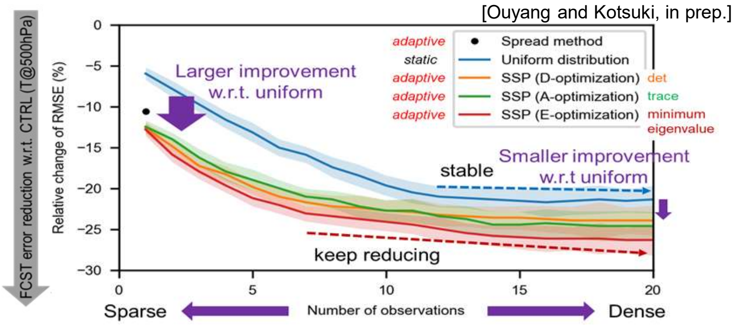
to observe additional profiles of atmosphere (radiosonde-like obs)

- **Experimental Setting (2 yrs)**
  - 20-member LETKF coupled with SPEEDY
  - Assimilating radiosonde-like osbs at obs stations every 6h
- **Experiments**
  - CTRL: radiosonde-like network
  - UNIF: allocate additional obs uniformly (Kotsuki et al. 2020; QJRMS)
  - SSP: D, A or E optimizations based on T at 500hPa



Sensitivity to # of obs

[Ouyang and Kotsuki, in prep.]



SSP outperforms the uniform distribution, when observations are sparser.

Placed observations (# obs = 4)

SSP-E (max (the smallest eigenvalue of **F**)) shows more diversity
➔ Maybe, it tries to find obs sensitive to lower modes.