WAVES TO WEATHER

The Weak Temperature Gradient approximation as a balance principle for convective-scale data assimilation

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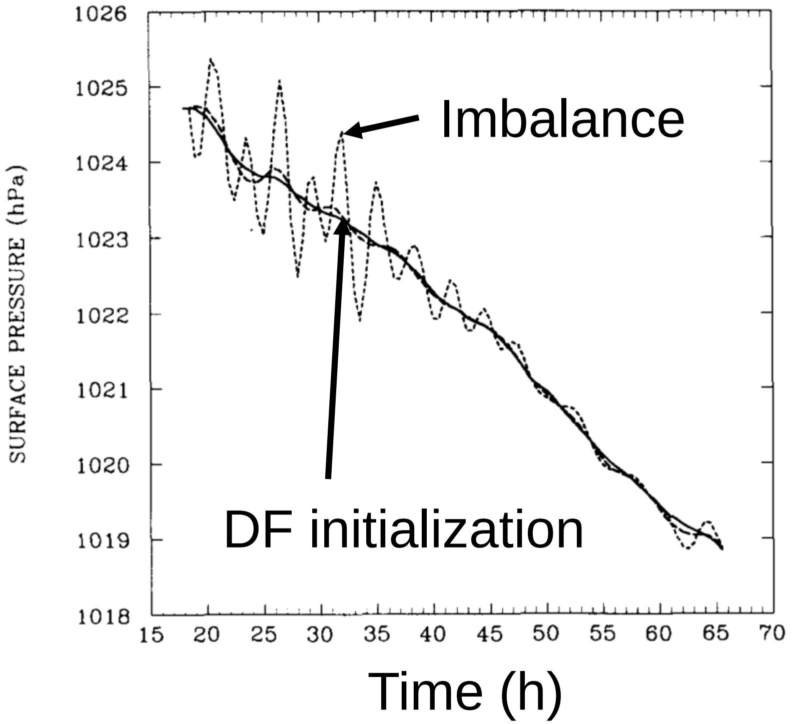
Balance and Imbalance

Atmospheric flows in nature stay close to "balanced" states
- Synoptic scale balanced states are:
  - slow, rotational
  - *e.g.* geostrophic, hydrostatic, (Charney nonlinear, ...)

Data assimilation increments may be unbalanced
- Unbalanced motions are:
  - fast, divergent
  - *e.g.* gravity waves, (acoustic waves ...)

Methods to reduce imbalance
  - realistic covariance model (statistics, EnKF),
  - impose balance in increments (*e.g.* hydrostatic),
  - avoid large initial tendencies (*e.g.* incremental updates)
  - damp fast motions (*e.g.* NNMI, digital filter)

Surface pressure at 0°W, 68°N



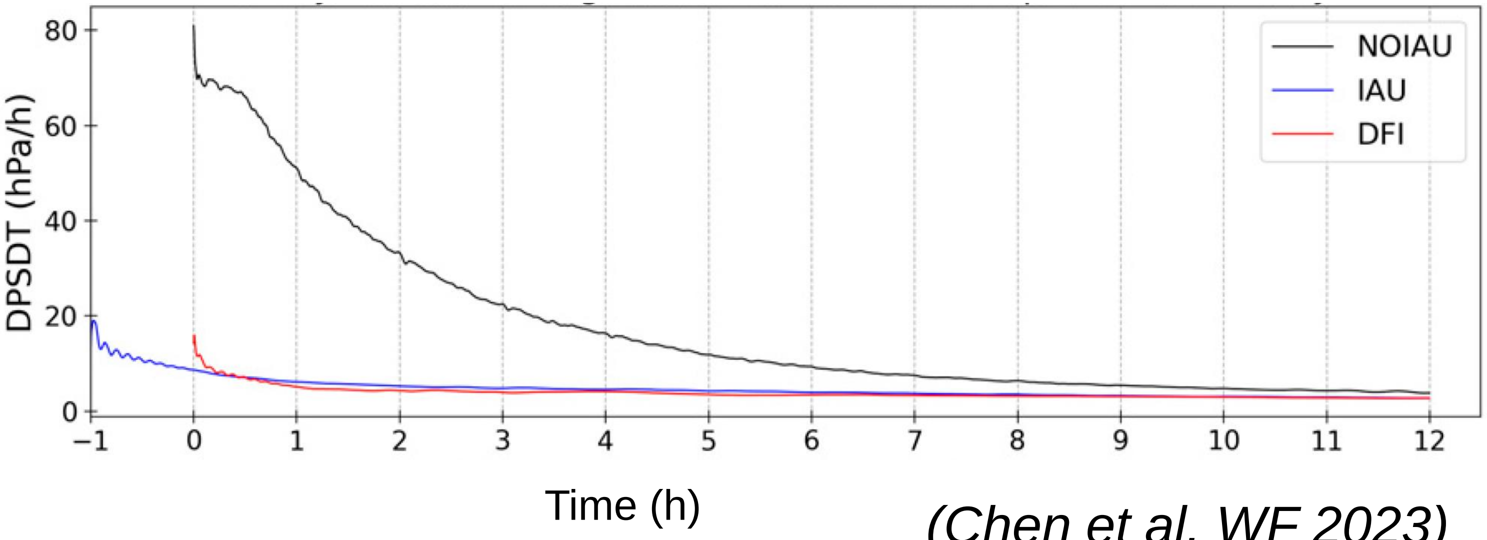
(Fillion et al. Tellus 1995)

Are convective scale motions "balanced"?

- Data assimilation does seem to produce unphysical states
- Spurious motions are:
  - fast, *e.g.* large time tendencies
  - small scale, *e.g.* gravity waves
  - apparent in diagnostics like surface pressure tendency

- "Imbalance" reduced by:
  - less localization in EnKF
  - incremental updates
  - digital filters

Surface pressure tendency



(Chen et al. WF 2023)

Spurious convection

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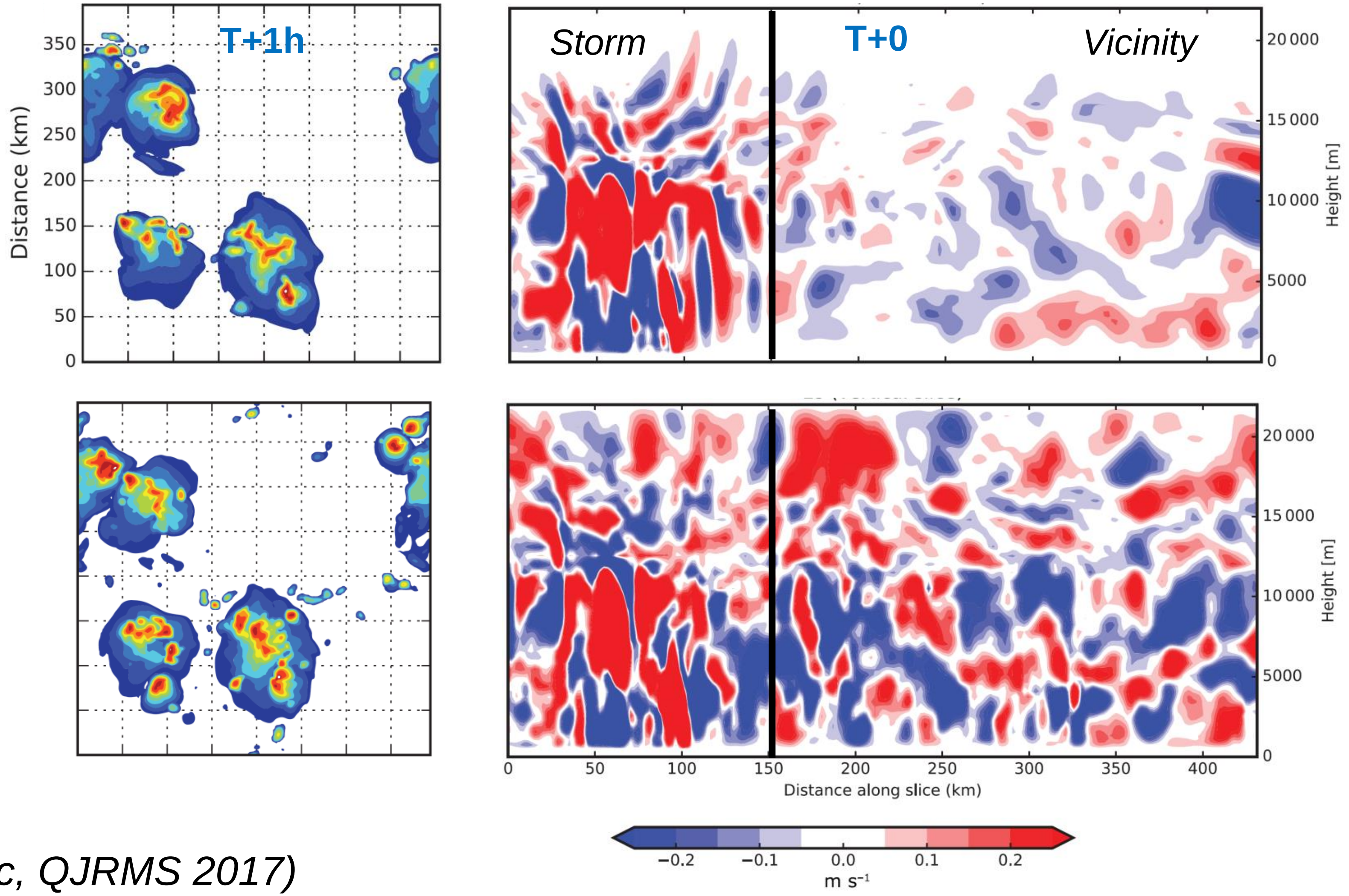
Simulated reflectivity

Vertical velocity along cross section

Nature run

EnKF with strong localization produces spurious convective cells



Source of spurious cells is gravity waves – not cold pools or other changes in structure of storms

(Lange, Craig, Janjic, QJRMS 2017)

State of the art regarding balance on convective scales(?)

1. Synoptic scale balance relations do not apply on convective scale
   - Enforcing synoptic balance conditions on convective scale motions degrades analysis (*e.g.* Bannister 2021)

2. Unphysical noise produced by DA increments has similar character to synoptic scale counterparts
   - Similar diagnostics can measure it
   - Similar methods can reduce it

Conclusion: Seek balance principle that eliminates fast motions and gravity waves

Timescales fast and slow

"Balanced" motion (advection): $\tau_{adv} \sim \frac{L}{V}$

Rotation (inertial frequency): $\tau_{rot} \sim \frac{1}{f}$

Gravity waves $\tau_{gw} \sim \frac{L}{NH}$

$$\frac{\tau_{rot}}{\tau_{adv}} = \frac{V}{fL} = Ro$$

$$\frac{\tau_{gw}}{\tau_{adv}} = \frac{V}{NH} = Fr$$

Mid-latitude synoptic: $Ro \sim Fr \rightarrow 0$ → Quasi-geostrophic approx.

Convective: $Ro \geq 1, Fr \rightarrow 0$ → Weak temp. gradient approx.

(Klein ARFM 2010)

WTG – Weak Temperature Gradient dynamics

Assumption of weak temperature gradients: $\theta = \bar{\theta}(z) + Fr^2\theta'(x, y, z, t)$

Dominant balance in thermodynamic equation: $Fr^2(\partial_t + u\partial_x + v\partial_y + w\partial_z)\theta' + w\partial_z\bar{\theta} = Q_\theta$

$$w_{WTG} \equiv \frac{Q_\theta}{\partial_z\bar{\theta}}$$

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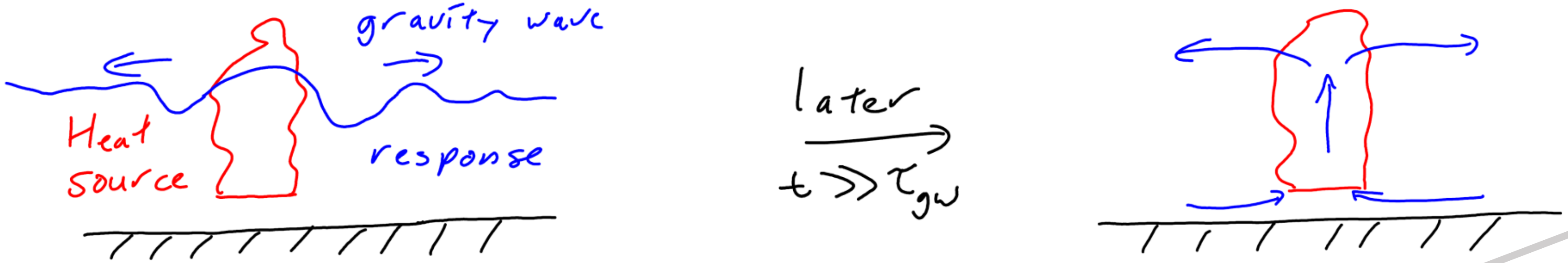


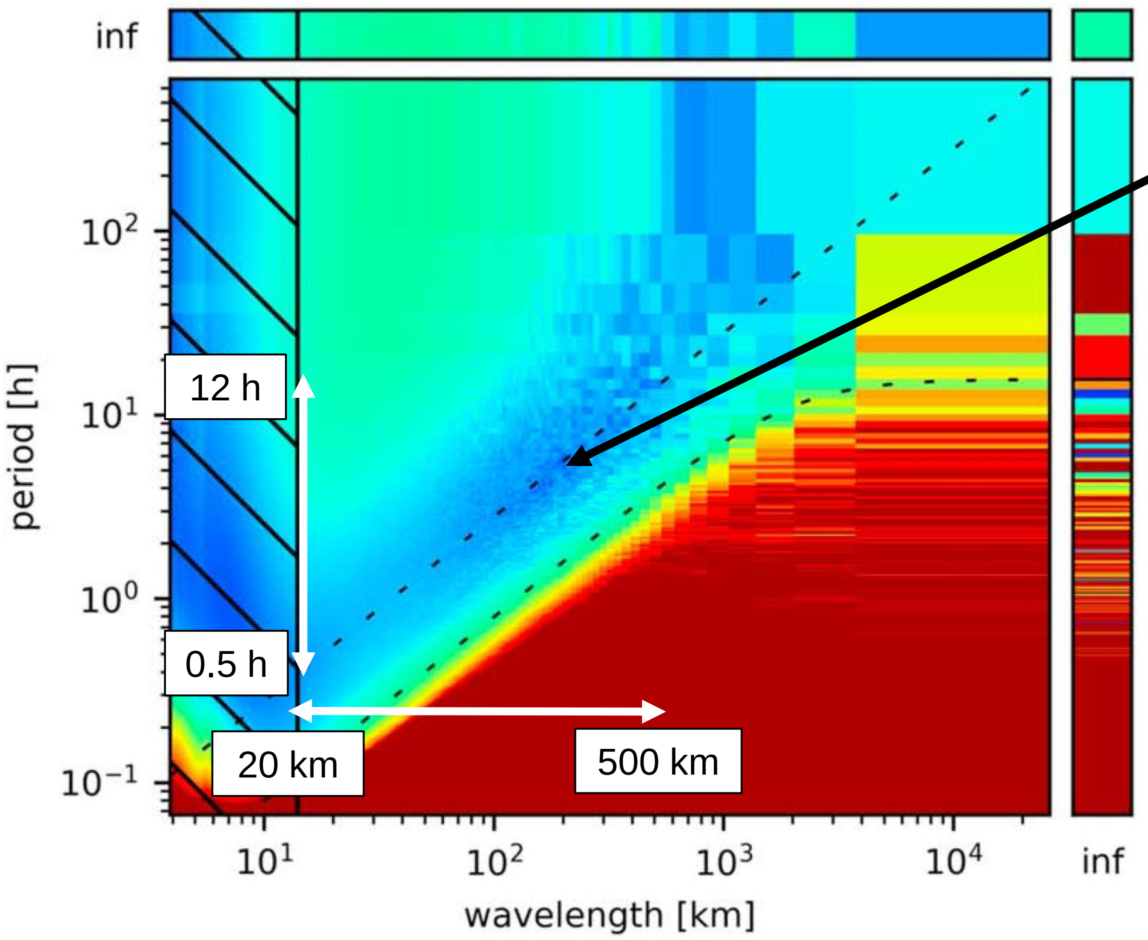
$$w_{WTG} \equiv \frac{Q_\theta}{\partial_z\bar{\theta}}$$

- **Unbalanced flow** is transient gravity wave response
- Small *Fr* results when heat source evolves slowly compared to time required for gravity waves to exit region of interest

- **Balanced flow** is a divergent response to diabatic heating/cooling
- No horizontal temperature gradients are produced, since adiabatic warming/cooling balances diabatic sources

How good is the WTG approximation?

$$Fr^2 \approx \frac{(\partial_t + u\partial_x + v\partial_y + w\partial_z)\theta'}{w\partial_z\bar{\theta}}$$



Estimate Fr^2 from km-scale NWP model ...

***Fr*² ~ 0.5**

→ not as good as geostrophic balance on synoptic scales (*Ro* ~ 0.2)

Three contributions:

1. Forcing changes too rapidly
   (Recall: want τ_{source} >> *L/NH* ~ 100 km/(10^{-2} s^{-1}·10 km) ~ 20 min)
2. Orographic gravity waves
3. Away from forcing – layered anisotropic stratified turbulence?

(Craig and Selz GRL 2018)

Diagnosis of imbalance

Departure of *w* from WTG solution is measure of imbalance

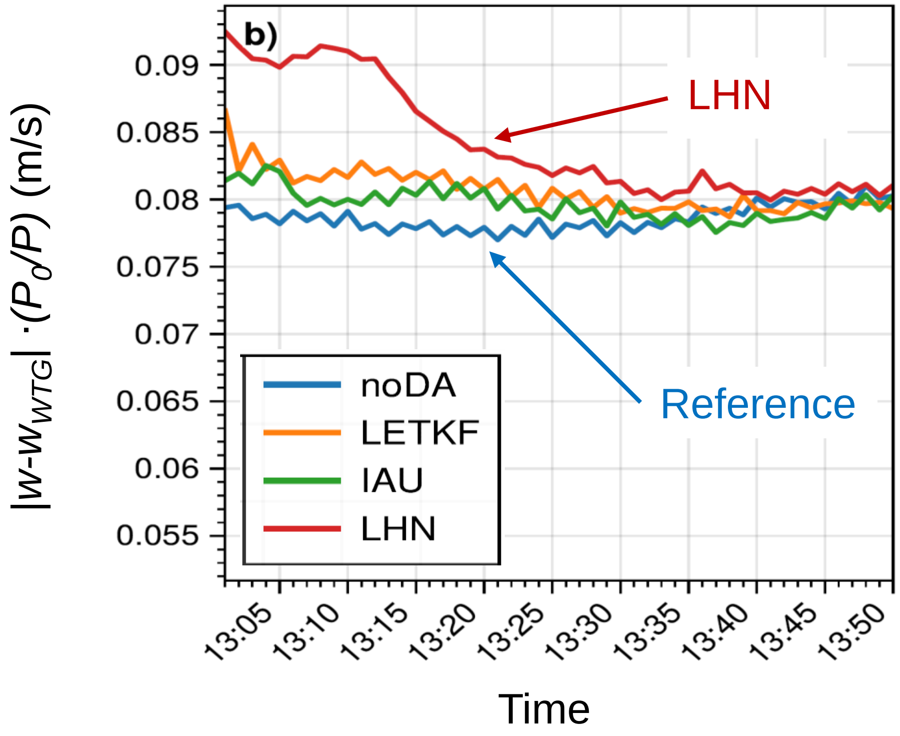
$$\text{Imbalance} = \overline{|w - w_{WTG}|}$$

Example:

- ICON-D2-KENDA forecasts over Germany
- Reference simulation and three DA experiments
  - LHN – Latent Heat Nudging
  - LETKF – ensemble Kalman Filter
  - IAU – LETKF with incremental updates
- WTG departure normalized by amount of convection, *e.g.* precipitation rate

Full presentation and comparison with other imbalance diagnostics
→ Theresa Diefenbach today 15:00

WTG Departure (normalized)



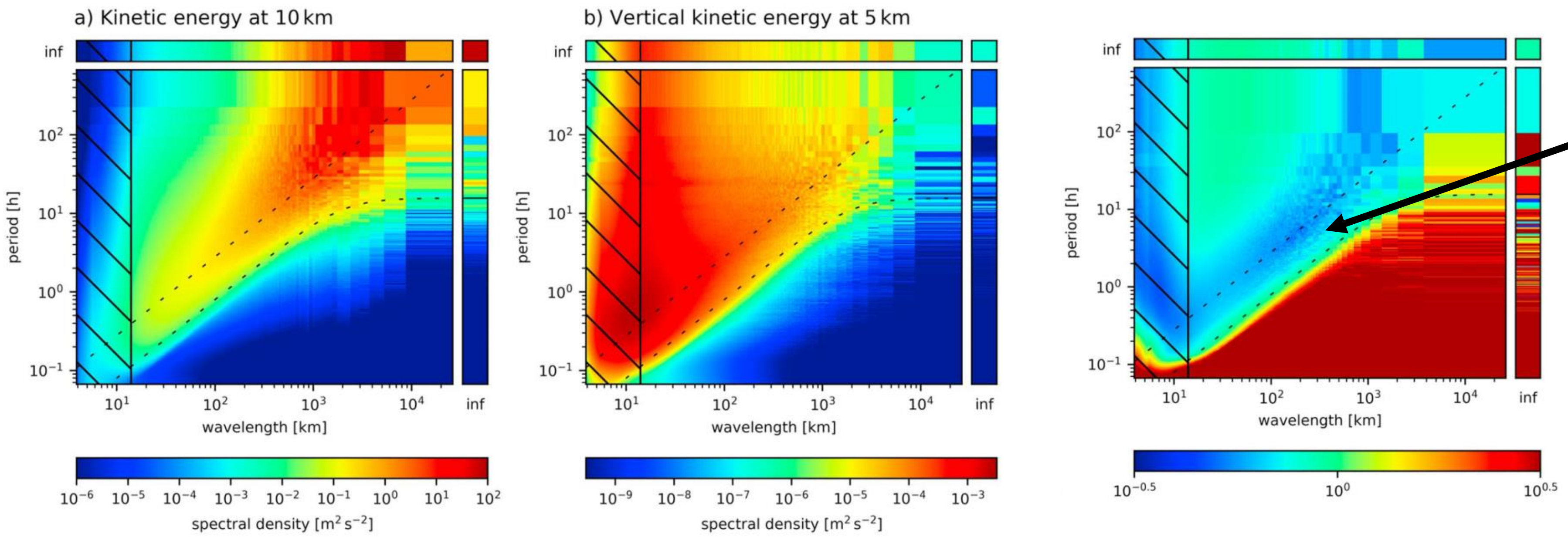
Conclusions

- No accepted balance principle for convective scale
- DA increments produced unwanted fast motions that have similar characteristics to synoptic scale unbalanced motions

- WTG is a natural approximation for "slow" motions on convective scale
- Separation between balanced and unbalanced motions not as well defined as for synoptic scale

- Departure from WTG balance may be useful diagnostic for convective scale imbalance
- Penalize departure from WTG balance to reduce imbalance?

How good is the WTG approximation?



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