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Studies on breeding-driven adaptive Data Assimilation applied to relevant Low Order Models

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Low Order Models (LOMs) allow to systematically explore many relevant issues connected with Numerical Weather and Ocean Prediction, thanks to their low computational and memory requirements. In this context, we investigated breeding-driven adaptive Data Assimilation (DA) strategies with different DA approaches, exploiting an open-source suite in Python. Two LOMs have been considered: Lorenz '63 (L63) and Lorenz-Emanuel '96 (LE96). In a first phase, mainly on L63, an "operational NWP-like" framework has been implemented, and different assimilation-prediction cycles have been tested. Adaptive DA strategies have been investigated mainly with the spatially distributed LE96. Best performances resulted for the Ensemble Kalman Filter approach, by adding the constraint of not assimilating over the same site in the following time-steps. This suggests the opportunity of including, in further studies, a space-time correlation analysis to improve the adaptive assimilation-sites selection strategy.

LOW ORDER MODELS

In the words of Edward Lorenz:

"[…] if the equations were sufficiently simplified, perhaps to the point where they could not produce good weather forecasts, but where they still might qualitatively reproduce some features of the general circulation […]", From Ref. [1]

A recipe for the construction of a Low-Order Model (LOM):

i) include the "needed" physics: non-linear terms, linear terms, constant terms

- simplify the original equations
- construct from scratch by summing suitable terms

$$\partial X_i/\partial t = \sum_{j,k} A_{ijk}(X_j, X_k) + \sum_j B_{ij}(X_j) + C_i$$

N variables X_i
L levels
M horizontal grid points

ii) horizontal and vertical discretization

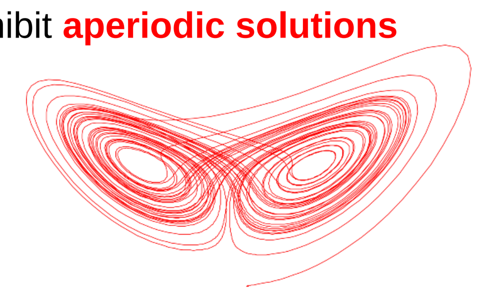
$$X_l = \sum_{m=0}^{M_l - 1} X_{lm}\phi_{lm}$$

iii) orthogonal functions expansion

- **Conservative LOMs** are more appropriate for short-term analyses
- **Forced dissipative LOMs** can be used for long-term simulations, as needed for comparison with "statistical weather prediction": these latter may exhibit aperiodic solutions

L63 model — lowest order terms of Saltzman's expansion

$$\dot{X} = -\sigma(X - Y)$$
$$\dot{Y} = rX - Y - XZ$$
$$\dot{Z} = b(XY - Z)$$



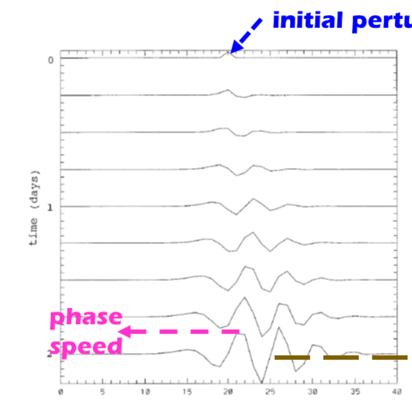
See Refs. [2, 3]

LORENZ-EMANUEL '96 LOM

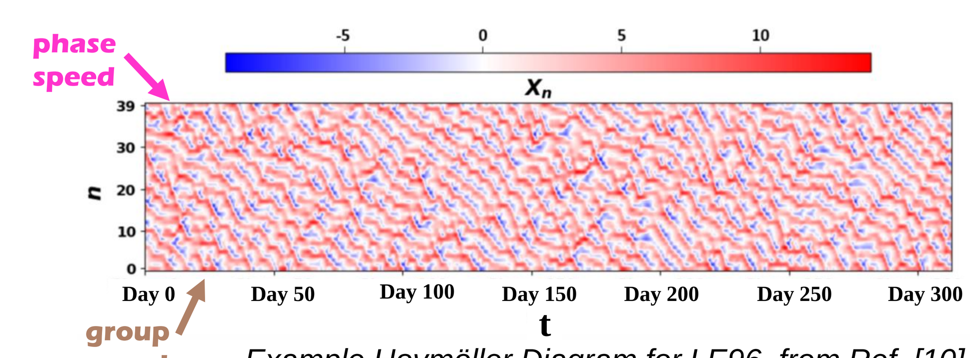
LE96 model

$$\frac{dX_n}{dt} = -X_{n-2}X_{n-1} + X_{n-1}X_{n+1} - X_n + \Phi, \quad n = 0, 1, 2, \ldots, K.$$

advection; damping; "external" forcing; periodic boundary conditions

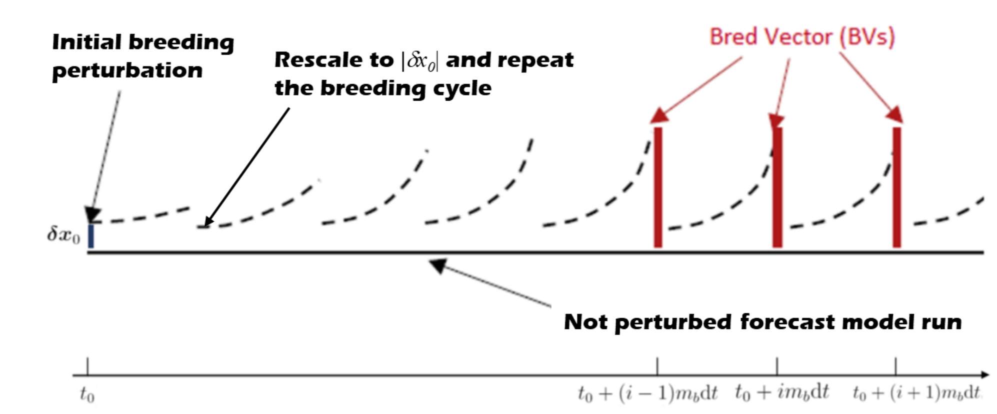


See Refs. [4, 5, 6]



Example Hovmöller Diagram for LE96, from Ref. [10]

BREEDING



See Ref. [7]

B-DADA: BREEDING-DRIVEN ADAPTIVE DATA ASIMILATION

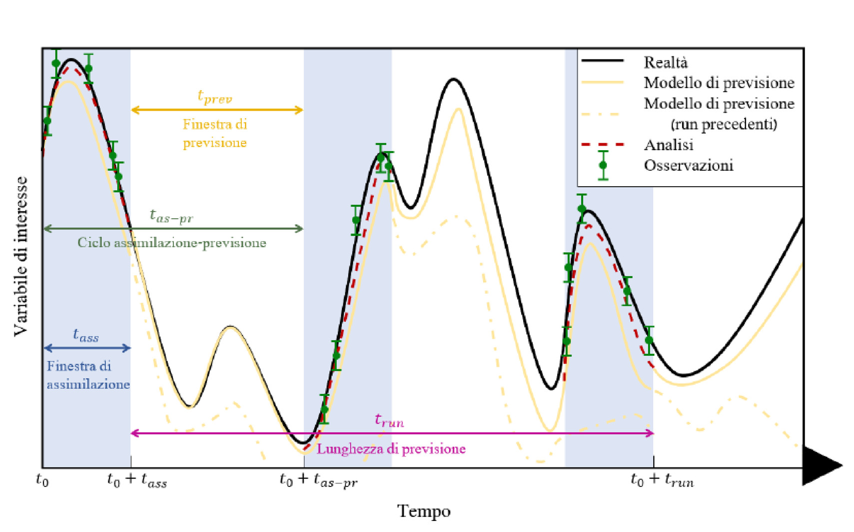
assimilate "when" and "where" the BV growth rate is greater

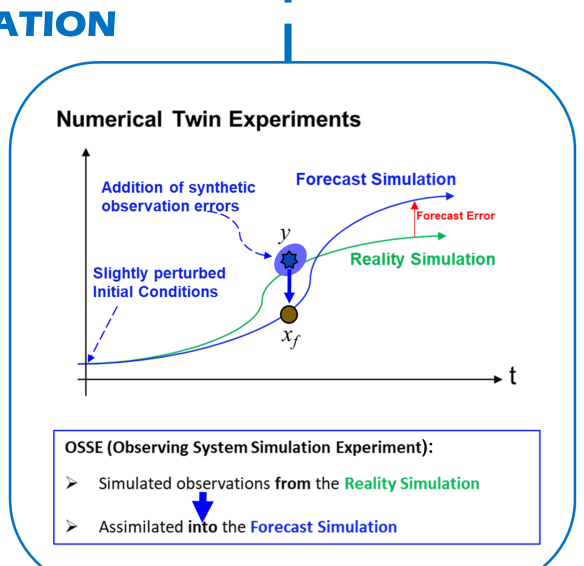
$$g(t) = \frac{1}{m_b}\ln\left(\frac{\|\delta x^t_{m_b}\|}{\|\delta x^t_0\|}\right)$$ **BV growth rate**

See Refs. [6, 7, 8, 9]

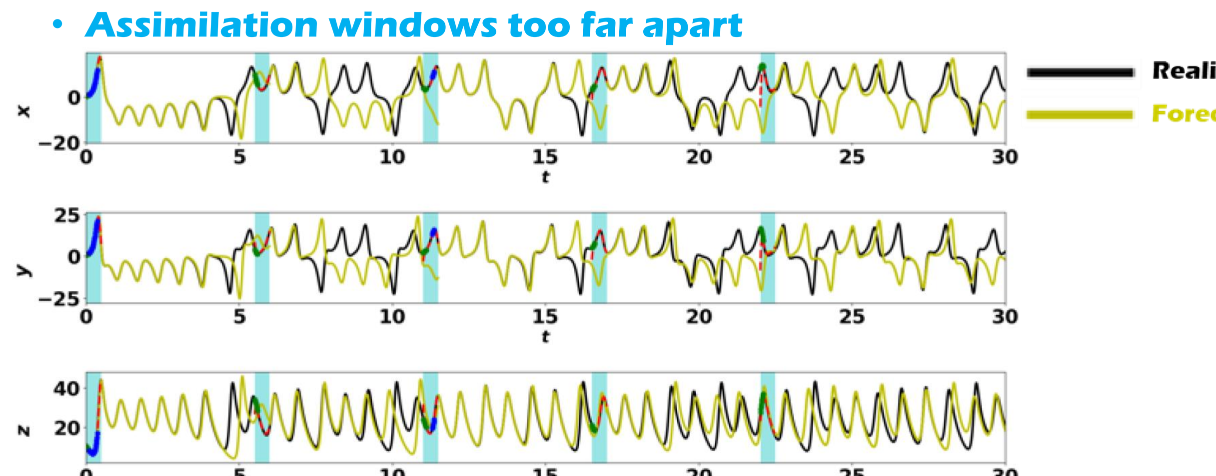
RESULTS (from Master Thesis of Alberto Baldi [10] developed by using the PyDA suite of DA algorithms in Python [11])

PHASE 1: BUILD AN OPERATIONAL-LIKE "FORECASTING" FRAMEWORK WITH L63 MODEL AND FIRST TESTS OF BREEDING-DRIVEN DATA ASSIMILATION

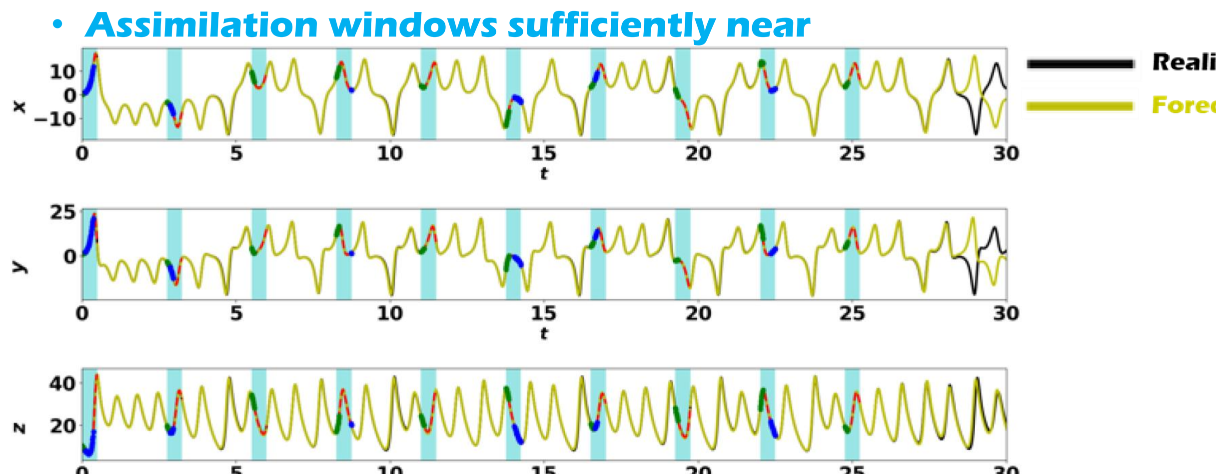




- **Assimilation windows too far apart**



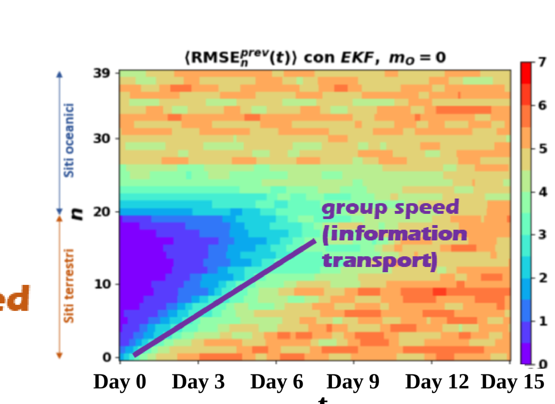
- **Assimilation windows sufficiently near**



PHASE 2: BREEDING-DRIVEN ADAPTIVE DATA ASSIMILATION WITH THE "LAND-OCEAN" LE96 MODEL

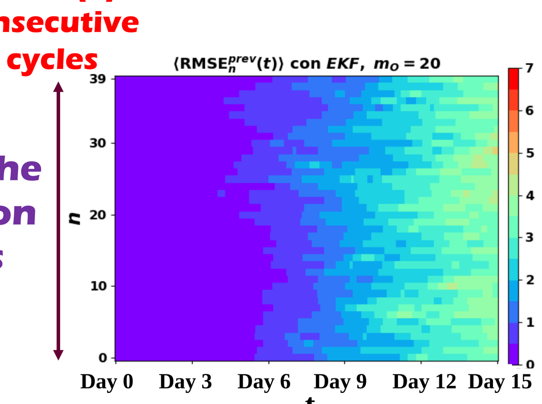
Ocean, 20 sites: no observations

Land, 20 sites: all with assimilated observations



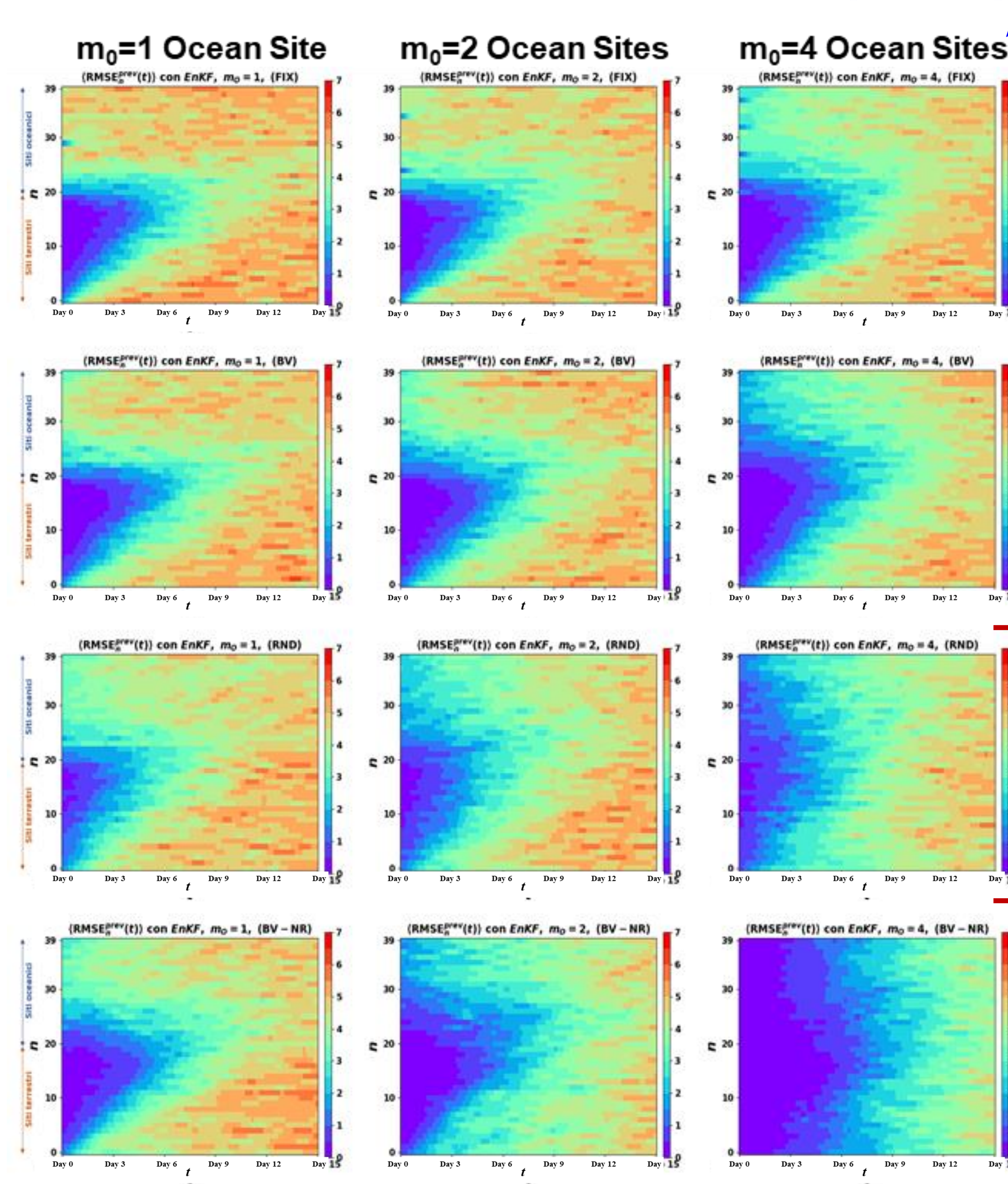
Forecast Error: RMSE (t) average over 100 consecutive assimilation-forecast cycles

Assimilating the observations on all the 40 sites



Ensemble Kalman Filter (EnKF)

Assimilation on: 20 Land sites and m_0 Ocean sites



Not adaptive DA strategy:

FIX: fixed positions

m_0 Ocean sites on fixed equi-spaced positions

Adaptive DA strategies:

BV: Bred Vector driven

m_0 Ocean sites with max BV growth rate

RND: Random

m_0 Ocean sites randomly selected

BV-NR: Bred Vector driven with No Repetition

m_0 Ocean sites with max BV growth rate, but no repeated assimilation on same sites

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CONCLUSIONS

Study Phase 1: an "operational NWP-like" framework has been implemented and tested with the L63 LOM, investigating different assimilation-forecast time intervals. First tests of breeding-driven adaptive DA strategies have been performed on L63, searching for optimized DA times. **Study Phase 2**: different breeding-driven DA strategies have been investigated with the "Land-Ocean" LE96 LOM, searching for optimal DA sites. Tests have been performed with various DA techniques from the PyDA Python suite [11]: *i*) 3D-VAR, *ii*) Extended Kalman Filter (EKF), *iii*) Ensemble Kalman Filter (EnKF). **Main Results**: with all the DA techniques, the best breeding-driven adaptive strategy resulted to be the one augmented by the constraint of not assimilating over the same site in the following time-steps (No-Repetition strategy). The DA technique that displayed the greatest improvements has been the EnKF. This is a relevant result given that EnKF is widely used in the real meteorological and oceanographic operational context. The better performance of the No-Repetition strategy suggests to investigate the opportunity of including, in further studies, a space-time correlation analysis to improve the adaptive assimilation-sites selection strategy.