Weakly Constrained LETKF for Convective-Scale Data Assimilation

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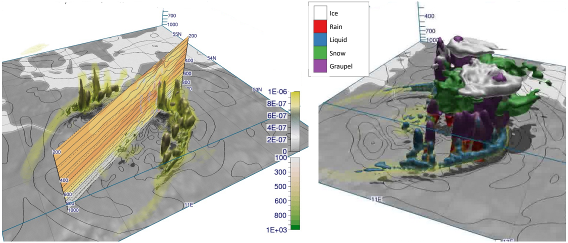
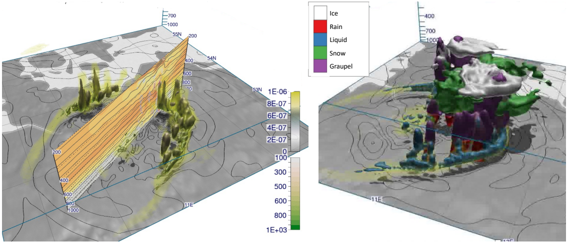
Convective/Storm scale application



Met3D

- High resolution NWP models of atmosphere that incorporate our knowledge of the dynamics and physics.

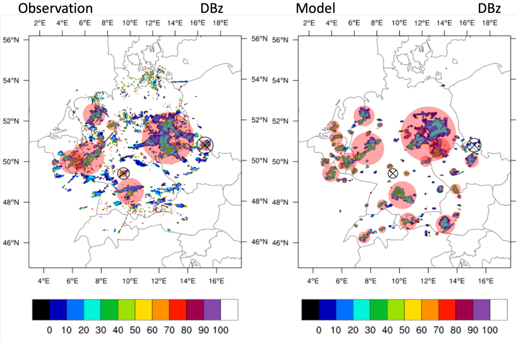
Convective/Storm scale application



Met3D

- High resolution NWP models of atmosphere that incorporate our knowledge of the dynamics and physics.
- In addition to dynamical variables, prognostic hydrometeors variables (rain, graupel, snow, ...) at all grid points

Uncertainty of geophysical models



- These models are not perfect. We need to specify background uncertainty including model error uncertainty.
- Forecast error for convective storms is often location and amplitude error.

Data assimilation algorithms

- Both ensemble and variational DA methods in use operationally (Gustafsson et al. 2018).
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- But also observational error covariance for radar data (Waller et al. 2019, MWR; Zeng et al. 2021, AMT).

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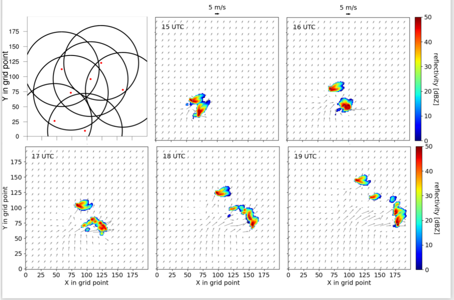
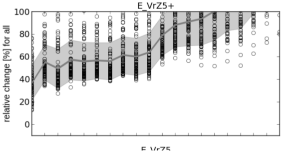
- But also observational error covariance for radar data (Waller et al. 2019, MWR; Zeng et al. 2021, AMT).
- Overview of challenges in Gustafsson et al. 2018 and Bannister et al. 2019, QJRMS.

A problem on convective scale

Idealized setup for radar DA



Zeng et al. 2021: Assimilating radar radial wind and reflectivity data in an idealized setup of the COSMO-KENDA system, Atmospheric Research, 249, 105282, https://doi.org/10.1016/j.atmosres.2020.105282.



Analysis mass of all hydrometeors compared to truth during DA (Janjic and Zeng, Geophysical Research Letters, 2021).

— Some solutions —

- Physical properties/Conservation laws
- Weak formulation for mass/positivity constraints

QPEns algorithm

$$\mathbf{w}_k^{a,i} = \mathbf{w}_k^{b,i} + \arg\min_{\delta w^i} \frac{1}{2}[\delta\mathbf{w}^{i\,T}(\mathbf{P}^b)^{-1}\delta\mathbf{w}^i + \mathbf{f}^{i\,T}\mathbf{R}^{-1}\mathbf{f}^i]$$

subject to

$$\delta\mathbf{w}^i \geq -\mathbf{w}_k^{b,i}. \quad i = 1, \ldots N$$

$$\delta\mathbf{w}^i = \mathbf{w}_k^{a,i} - \mathbf{w}_k^{b,i}, \mathbf{f}^i = \mathbf{w}_k^{o,i} - \mathbf{H}_k\mathbf{w}_k^{b,i} - \mathbf{H}_k\delta\mathbf{w}^i - \bar{\mathbf{r}}_k^o.$$

Janjic, T., D. McLaughlin, S. E. Cohn, M. Verlaan, 2014: Conservation of mass and preservation of positivity with ensemble-type Kalman filter algorithms, Mon. Wea. Rev., 142, No. 2, 755-773.

Weak Constraint Convective scale

- DA on convective scales should update hydrometeors
- When DA algorithms update hydrometeors they clip negative values to zeros, modifying mass
- Janjic et al. 2014 show that is important to preserve both positivity and mass with DA algorithm when estimating variables that should be non-negative
- Here, we propose a fast, easy to implement modification of LETKF that is able to weakly preserve both properties of mass conservation for each hydrometeor variable and non-negativity.

Weakly Constrained LETKF

$$\min_{x} \quad J(\mathbf{x}) = J_b(\mathbf{x}) + J_o(\mathbf{x}) + J_m(\mathbf{x})$$

$$= \frac{1}{2}(\overline{\mathbf{x}}_k^b - \mathbf{x})\mathbf{P}_k^{b^{-1}}(\overline{\mathbf{x}}_k^b - \mathbf{x})^T + \frac{1}{2}\left[\mathbf{y}_k^o - \mathbf{H}(\mathbf{x})\right]\mathbf{R}_k^{-1}\left[\mathbf{y}_k^o - \mathbf{H}(\mathbf{x})\right]^T$$

$$+ \frac{1}{2}\left[\mathbf{m}_k - \mathbf{S}(\mathbf{x})\right]\mathbf{M}_k^{-1}\left[\mathbf{m}_k - \mathbf{S}(\mathbf{x})\right]^T$$

- $\mathbf{m}$ is a vector quantity, whose elements are the domainwise (global) integral of hydrometeors.
- $\mathbf{S}$ is operator which calculates the domainwise (global) integral for each of the microphysical spieces,

Constraint on mass is up to accuracy $\mathbf{M}_k$

$$\mathbf{M}_k = \frac{1}{N_{ens} - 1}\sum_{i=1}^{N_{ens}}\left[\mathbf{m}_k^* - \mathbf{S}\left(\mathbf{x}_k^{b(i)}\right)\right]\left[\mathbf{m}_k^* - \mathbf{S}\left(\mathbf{x}_k^{b(i)}\right)\right]^T$$

Weakly Constrained LETKF

For mass:

$$\overline{\mathbf{x}}_k^{a,M} = \quad = \overline{\mathbf{x}}_k^b + \mathbf{P}_k^{a,M}\mathbf{H}^T\mathbf{R}_k^{-1}\left(\mathbf{y}_k^o - \overline{\mathbf{y}}_k^b\right) + \frac{1}{N_{ens}-1}\mathbf{X}_k^{a,M}\left(\mathbf{S}\mathbf{X}_k^{a,M}\right)^T\mathbf{M}_k^{-1}\left[\mathbf{m}_k - \mathbf{S}\left(\overline{\mathbf{x}}_k^b\right)\right]$$

$$\mathbf{W}_k^{a,M} = \left[(N_{ens}-1)\mathbf{I} + \left(\mathbf{H}\mathbf{X}_k^b\right)^T\mathbf{R}_k^{-1}\mathbf{H}\mathbf{X}_k^b + \left(\mathbf{S}\mathbf{X}_k^b\right)^T\mathbf{M}_k^{-1}\mathbf{S}\mathbf{X}_k^b\right]^{-1},$$

Note:

1. Corrections to weights implemented locally
2. After analysis ensemble is calculated correction to mass of each member

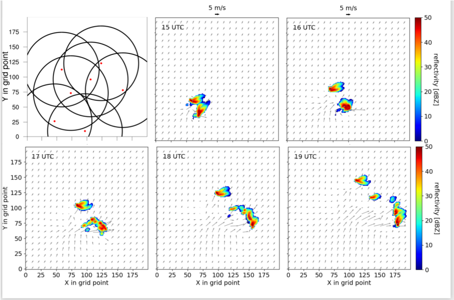
Weakly Constrained LETKF

For positivity:

- To avoid spurious convection (Aksoy et al., 2009) clear-air reflectivity data are assimilated, i.e. non-negative threshold value is set for very small reflectivities.
- Radar reflectivity data depend nonlinearly on hydrometeors. Further, reflectivity data (including clear-air reflectivity data) are available at radar observation locations, therefore not in every grid point of the model
- By assimilating additional clear-air reflectivity data, we are asking in the approximate weak sense that non-negativity is preserved in the analysis of hydrometeors

Experimental setup

Idealized setup for radar DA



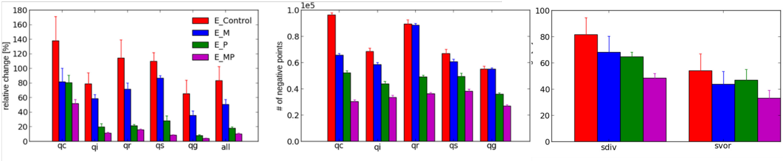
Zeng et al. 2021: Assimilating radar radial wind and reflectivity data in an idealized setup of the COSMO-KENDA system, Atmospheric Research, 249, 105282, https://doi.org/10.1016/j.atmosres.2020.105282.

- COSMO model with a 2-km horizontal resolution
- Efficient Modular VOlume scanning RADar Operator (EMVORADO, Zeng et al., 2014, 2016)
- Both radial wind and reflectivity data are assimilated
- Ensemble size is 80
- Assimilated observations are perturbation of nature run with Gaussian noise with a standard deviation of 5.0 dBZ and 1.0 m/s

Results: Impact of the different constraints

1. $E_{Control}$ Radar reflectivity and wind assimilated
2. E_M With mass constraint
3. E_P With positivity constraint (clear-air reflectivity data)
4. E_{MP} Both constraint

If clear-air reflectivity data are assimilated, a threshold value of 5 dBZ is set, that is, all reflectivity values smaller than 5 dBZ are set to 5 dBZ. If clear-air reflectivity data are not assimilated, all reflectivity values smaller than 5 dBZ are set to missing values.



Janjic, T. and Y. Zeng, 2021, Weakly constrained LETKF for estimation of hydrometeor variables in convective-scale data assimilation, Geophysical Research Letters, 48, e2021GL094962, https://doi.org/10.1029/2021GL094962.

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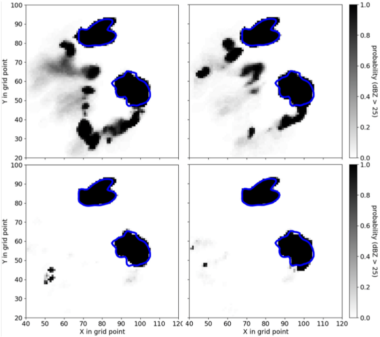
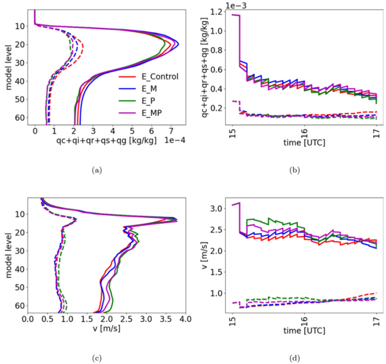
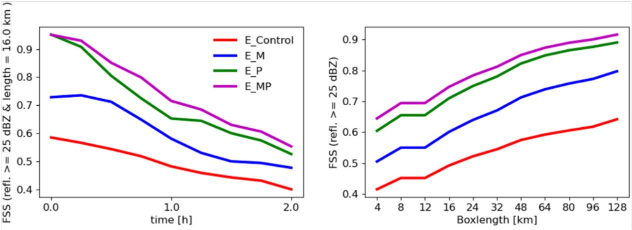


True storm plotted with blue line. From upper left to lower right: $E_{control}$, E_M, E_P, E_{MP}.



RMSEs calculated within storms only (full) and for full domain (dashed).

Accuracy



Accuracy of short term forecasts.

FSS score through time (left) and in dependence of grid box (right).

Conclusion

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- Some methods proposed require only minor changes to the already existing implementation.

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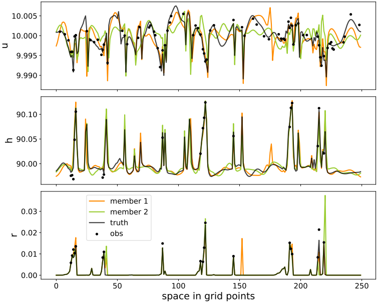
- Data assimilation for geophysical models that resolve many scales of motion and for observations of higher temporal and spatial density would require that we re-evaluate and improve on the methodology.
- We propose inclusion of conservation laws and physical constraints to tackle the difficult problem of convective scale data assimilation.
- By including mass and positivity, we improve on prediction of convective events.
- Some methods proposed require only minor changes to the already existing implementation.
- The inclusion of model error and observation error are still required.

Modified shallow water model

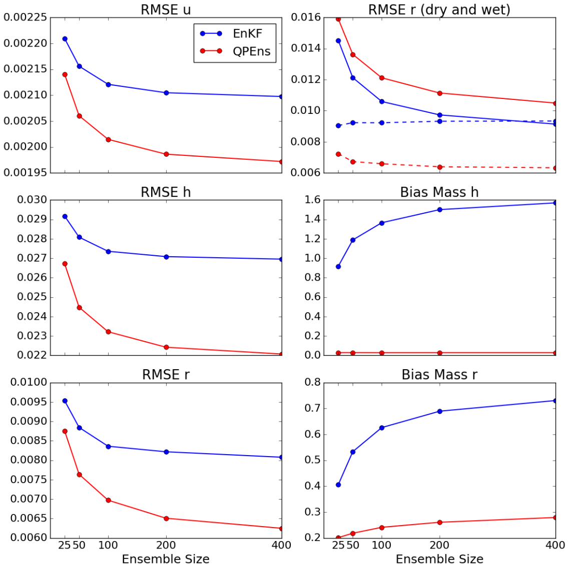
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial(\phi + \gamma^2 r)}{\partial x} = \beta_u + D_u\frac{\partial^2 u}{\partial x^2}, \phi = \begin{cases} \phi_c & \text{if } \ h > h_c \\ gh & \text{otherwise}, \end{cases}$$

$$\frac{\partial r}{\partial t} + u\frac{\partial r}{\partial x} = D_r\frac{\partial^2 r}{\partial x^2} - \alpha r - \begin{cases} \delta\frac{\partial u}{\partial x}, & \text{if } h > h_r \text{ and } \frac{\partial u}{\partial x} < 0 \\ 0 & \text{otherwise}, \end{cases}$$

$$\frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} = D_h\frac{\partial^2 h}{\partial x^2}.$$



Wuersch and Craig 2014: A simple dynamical model of cumulus convection for data assimilation research., Meteorol. Z., 23, 483-490.



Ruckstuhl and Janjic 2018: Parameter and state estimation with ensemble Kalman filter based algorithms for convective scale applications. Q.J.R. Meteorol. Soc.. 144:712, 826–841, doi:10.1002/qj.3257.

Application of QPEns to high dimensional systems

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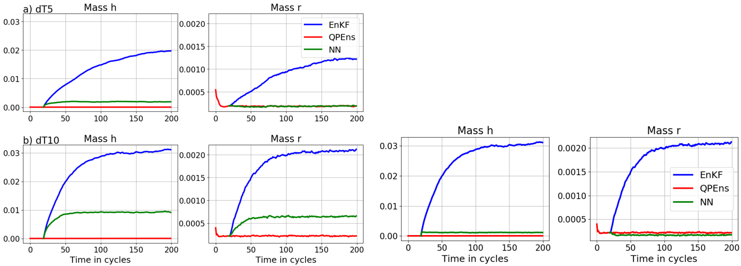
- High dimensional problem with third of the variables that need to be nonnegative and high update frequency
- We use the fact that constraints on mass and positivity are disjoint
- Active-set algorithm whose feature is to maintain feasibility with respect to the linear mass equality constraints on at each iteration, while at the same time using classical projection techniques to enforce positivity:

Alg1 still requires solving the Karush–Kuhn–Tucker (KKT) system
Alg2 exploits the low rank of the linear equality constraints (mass) and uses a well-known iterative approach to compute a possibly approximate solution while ensuring satisfaction of the constraint.

Janjić, T., Y. Ruckstuhl and P. Toint, 2021: A data assimilation algorithm for predicting rain, Q. J. R. Meteorol. Soc., 1949 –1963, doi:10.1002/qj.4004

Alternative approach via NN



Left: Deviation in total mass and total rain for NN trained on QPEns analysis obtained with 5 min and 10 min updates. Also EnKF and QPEns results are shown for comparison.

Right: For 10 minutes update if in addition penalty on mass is imposed during training.

Ruckstuhl, Y., T. Janjić, S. Rasp, 2021: Training a convolutional neural network to conserve mass in data assimilation, Nonlin. Processes Geophys., 28, **111–119, https://doi.org/10.5194/npg-28-111-2021.**