Learning model parameters from observations by combining data assimilation and machine learning

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MIDS

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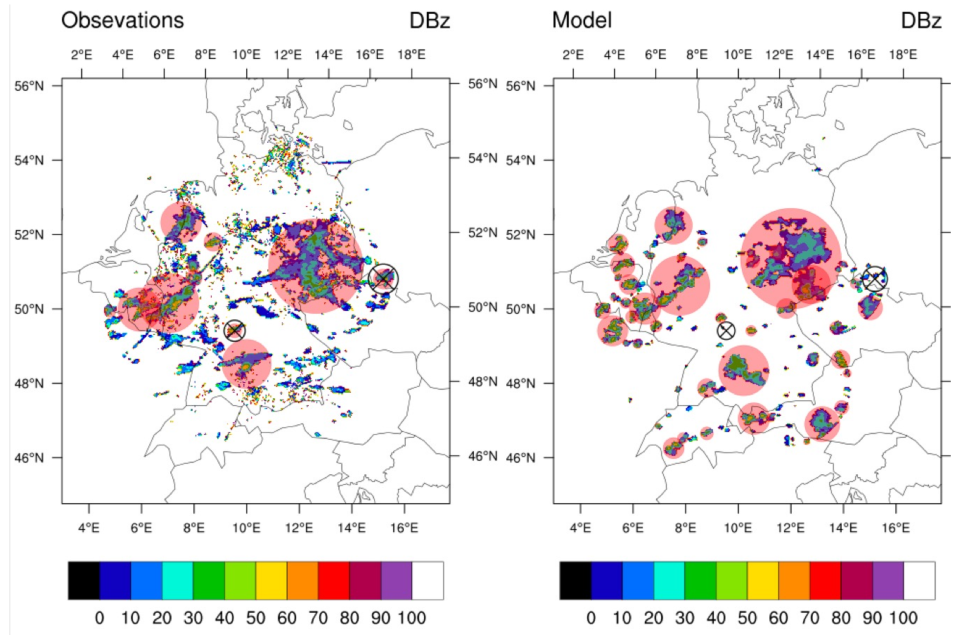
Motivation



Left: Verifying observations of radar reflectivity. Right: Radar reflectivity computed from one hour COSMO model forecast.

One of the reasons for model error is limited knowledge of model parameters.

Joint state and parameter estimation

Parameters are not observed. To learn parameters of a numerical model from observations

- Data assimilation
- Machine learning

Augment state vector **x** with parameters

Augmented data assimilation algorithm as EnKF requires stochastic model for parameters

$$\theta_k^{f,i} = \theta_{k-1}^{a,i} + \mathbf{D}_{k-1}\mathbf{C}^{\frac{1}{2}}\eta^i$$

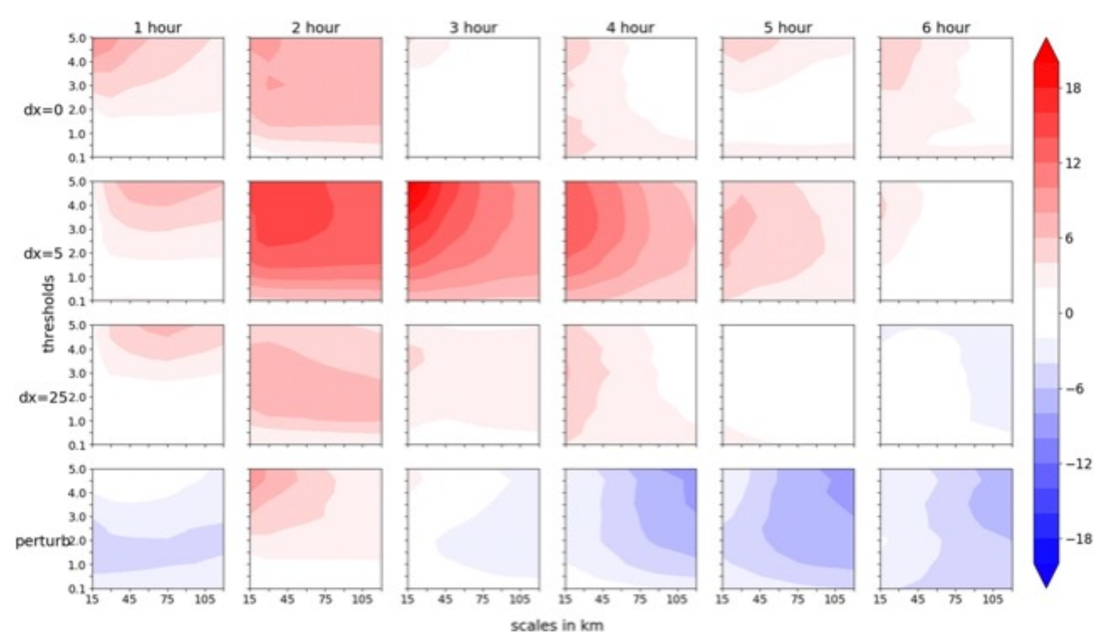
$\theta_{k-1}^{a,i}$ is the raw analysis value after applying the EnKF

$\theta_k^{f,i}$ the perturbed value that is passed to the model

$\mathbf{D}_{k-1}$ is a diagonal matrix that locally controls the ensemble spread

$\mathbf{C}^{\frac{1}{2}}$ is the error correlation matrix that specifies the correlations within parameter field

$\eta^i \sim \mathcal{N}(0, I)$ is the random realization of the stochastic model.



Sensitivity to stochastic model length scale: Precipitation FSS score in percentage withrespect to reference simulation when estimating roughness length jointly with the state

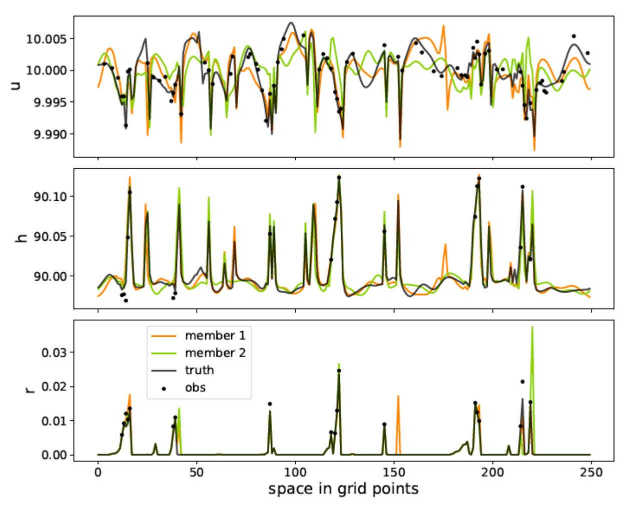
Accounting for model error by allowing uncertainty in parameters can reduce state error.

Modified shallow water model

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial(\phi + \gamma^2 r)}{\partial x} = \beta_u + D_u\frac{\partial^2 u}{\partial x^2}, \phi = \begin{cases} \phi_c & if\ \ h > h_c \\ gh & otherwise, \end{cases}$$

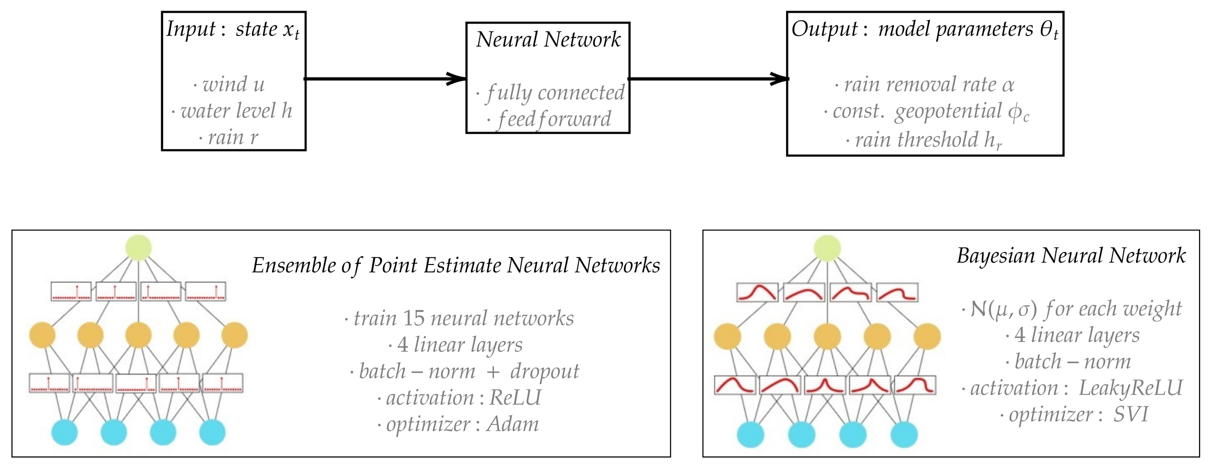
$$\frac{\partial r}{\partial t} \quad + \quad u\frac{\partial r}{\partial x} = D_r\frac{\partial^2 r}{\partial x^2} - \alpha r - \begin{cases} \delta\frac{\partial u}{\partial x}, & if\, h > h_r and \frac{\partial u}{\partial x} < 0 \\ 0 & otherwise, \end{cases}$$

$$\frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} = D_h\frac{\partial^2 h}{\partial x^2}.$$

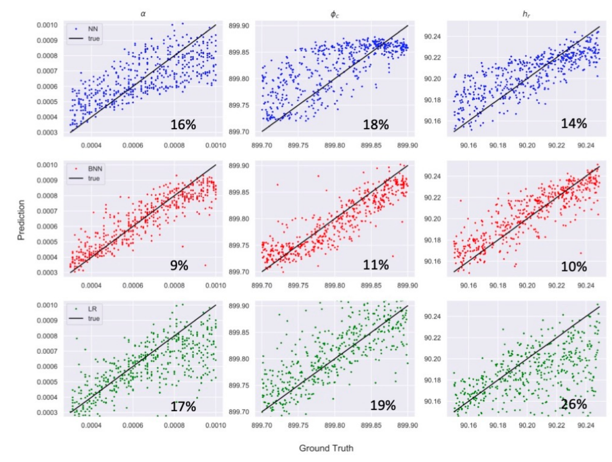


Model parameter estimates by combined data assimilation and machine learning approach

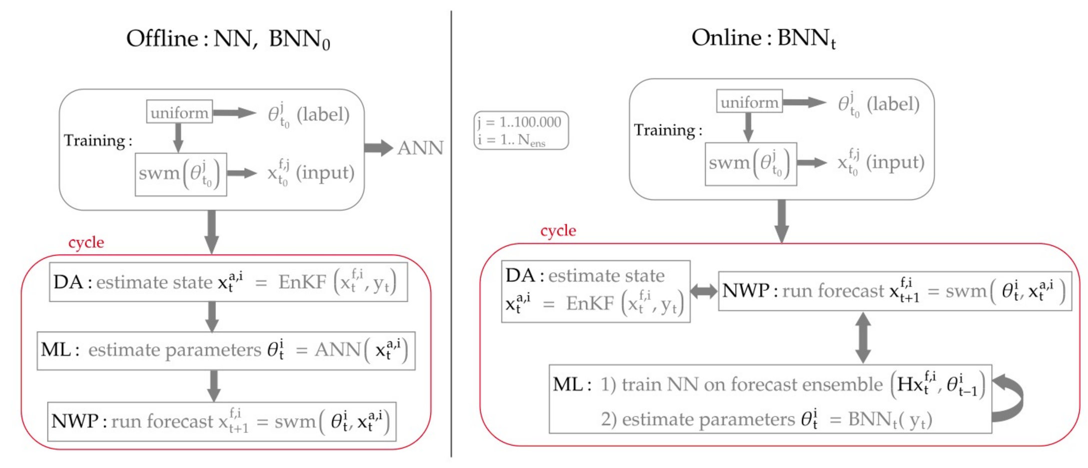
Two types of neural networks investigated for parameter estimation in idealized setup of Würsch and Craig, 2014: Point Neural networks (NNs) and Bayesian Neural Networks (BNNs)

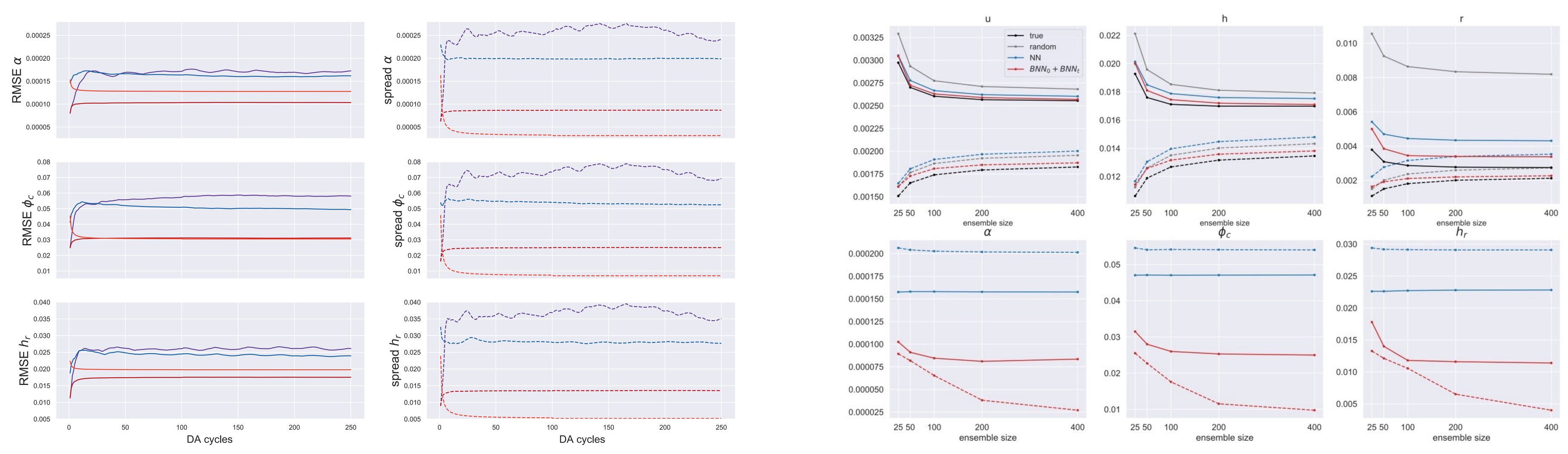


Method and Results

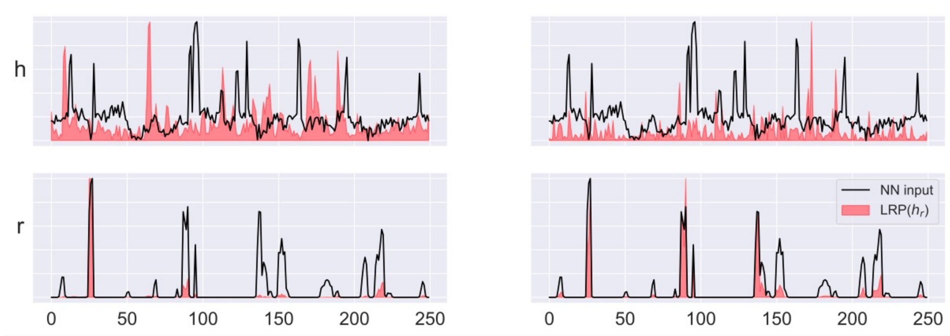


Output against corresponding truths, relative error and ideal line(black) of 500 samples





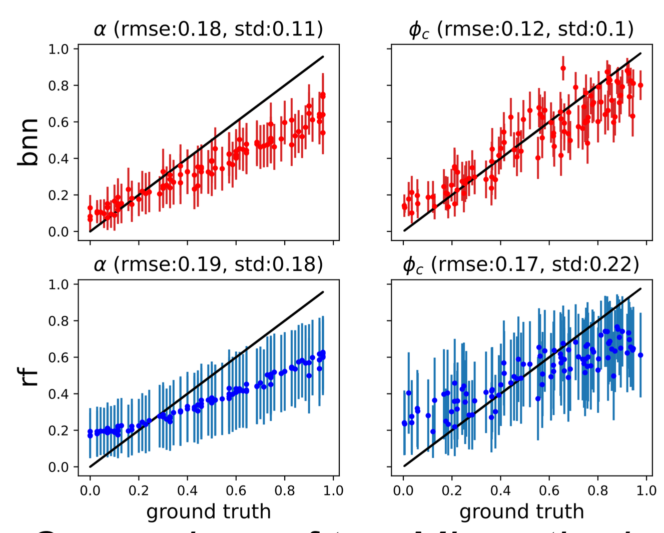
Left: RMSE and spread of parameters for BNN_0, NNs; BNN_t vs. time. Right: RMSE and spread for all variables and parameters of modified shallow water model vs. ensemble size.



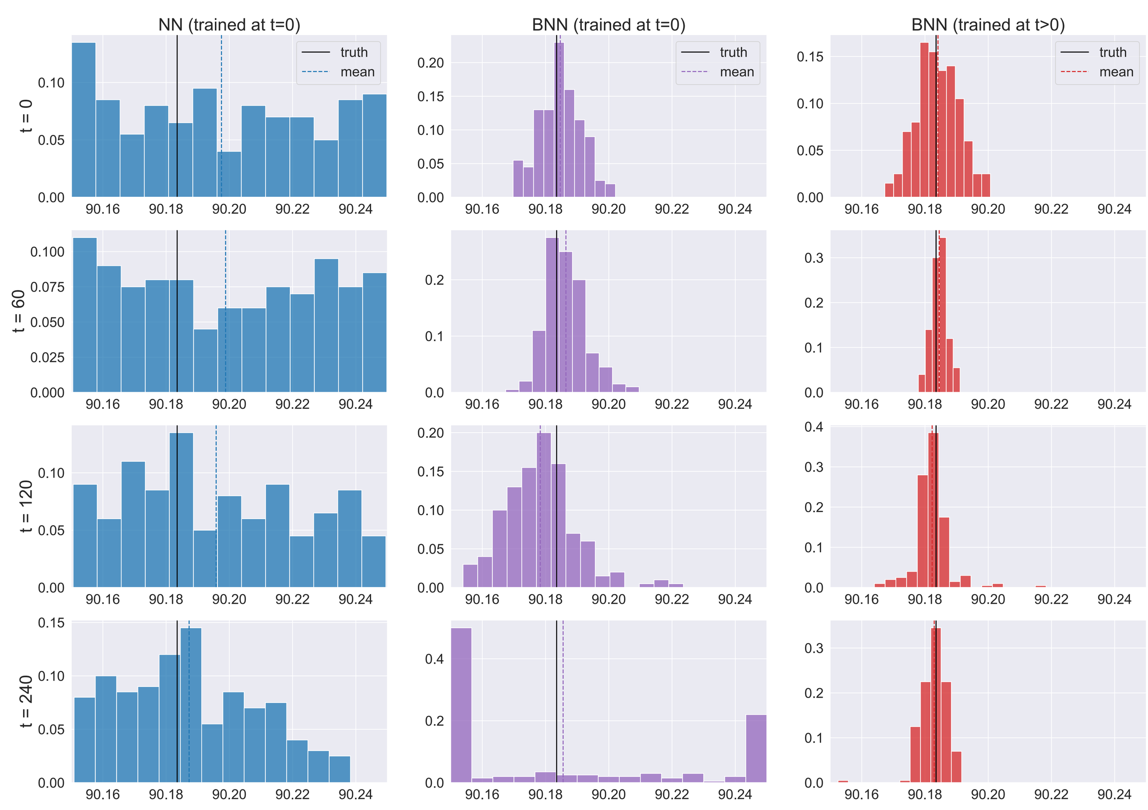
parameter	u	h	r
α	20%	57%	23%
ϕ_c	23%	61%	16%
h_r	21%	63%	16%

parameter	u	h	r
α	16%	59%	25%
ϕ_c	19%	44%	37%
h_r	21%	38%	41%

We follow Toms et al. 2020 to calculate Layer-wise relevance propagation (LRP) map for h_r in case Left: 3 parameters are estimated simultaneously. Right: LRP map when only h_r is estimated.



Comparison of two ML methods (BNNs and random forest) for estimates of a parameters and their uncertainty in a modified shallow water model (Legler et al. 2023).



Probability histograms of estimates of a parameter in a modified shallow water model from one single experiment for NN (left), BNN_0 (middle), and BNN_0+BNN_t (right) over time with 200 ensemblemembers (Legler and Janjic 2022).

Conclusions

- Estimating parameters can reduce prediction errors.
- If stochastic model for parameters can be made, EnKF can be used to objectively estimate parameters from data.
- Alternatively, BNNs and NNs can estimate model parameters and their relevant statistics by retrieving hidden stochastic model while data assimilation can provide data-sets for training/inference from sparse and noisy observations.

References

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