Formulation of 3D- and 4D-hybrid ensemble covariances in Météo-France global data assimilation

Loïk Berre, Etienne Arbogast, Nicole Girardot

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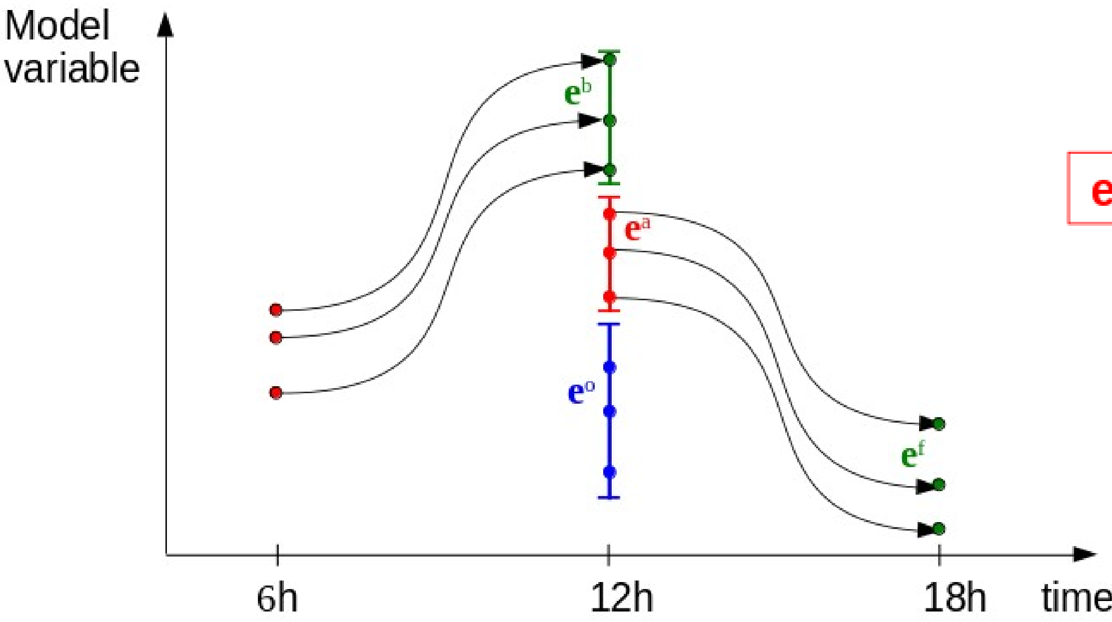
Simulation of analysis & forecast errors using EDA (50 members) for ARPEGE (global model at MF)



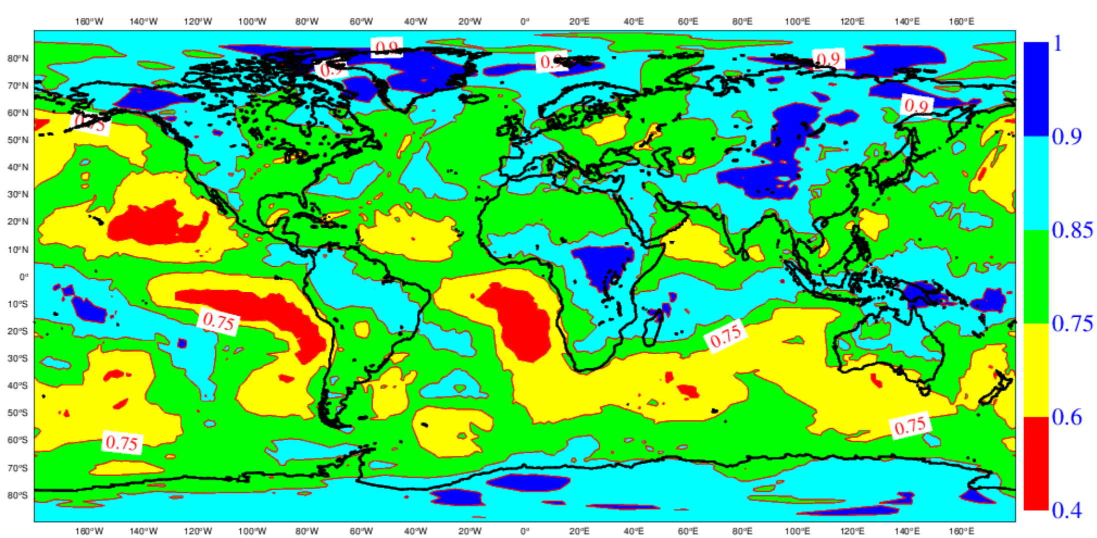
$e^a = (I-KH)e^b + Ke^o$

$e^f = Me^a + e^m$

with $e^b = e^{f-}$
and $e^o = R^{1/2}\,\eta$ (random draws of R)

Each of the 50 EDA members (~ 40 km) uses 4D-Var with obs perturbations & inflation of forecast perturbations.

Provides **time-dependent wavelet B** to high resolution 4D-Var, with 2 minims at 90km/40km resolution ; NL forecast at 5 km over France.



Vertical correlations of temperature background errors between 850 & 870 hPa

Such wavelet-based correlations are **isotropic horizontally**.

(e.g. Berre et al 2015)

Anisotropic ensemble 3D covariances, in 4D-Var with OOPS

3D covariances directly sampled by the ensemble and localised : $B_0^e = X_0^{b'} X_0^{b'T} \circ L$

(e.g. Buehner 2005, Clayton et al 2013)

- More direct use of ensemble information provided by the **50 EDA members**, with **flow-dependent anisotropies**.
- **Horizontal localisation** length-scale varies between 150 km (low layers) and 1,000 km (model top).
- **Vertical localisation** length-scale = 0.5 (in log hPa).
- Localisation scales are common to 5 model variables and isotropic.

=> **Transformation ensuring similar scales and isotropy** (wind, ln Ps) for localisation (Berre et al 2017) :

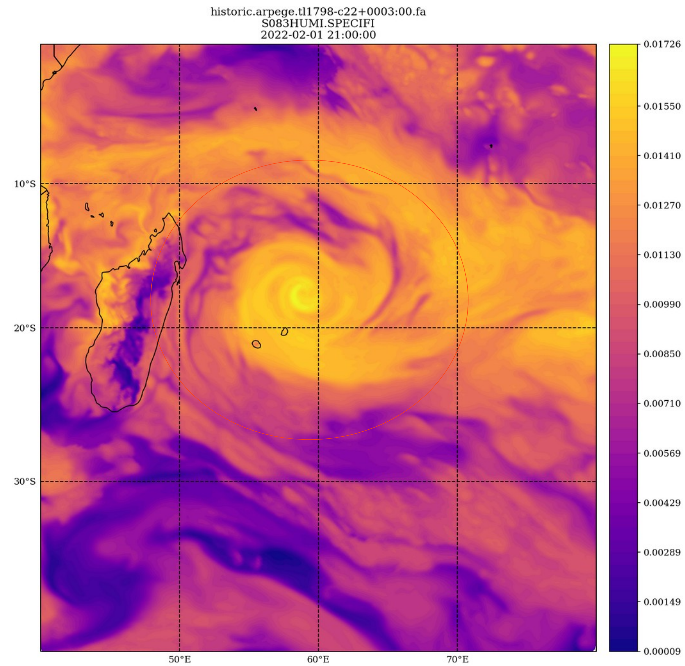
$$\widehat{X}_0^{b'} = V X_0^{b'}$$

$$B_0^e = V^{-1} [\widehat{X}_0^{b'} (\widehat{X}_0^{b'})^T \circ L] V^{-T}$$

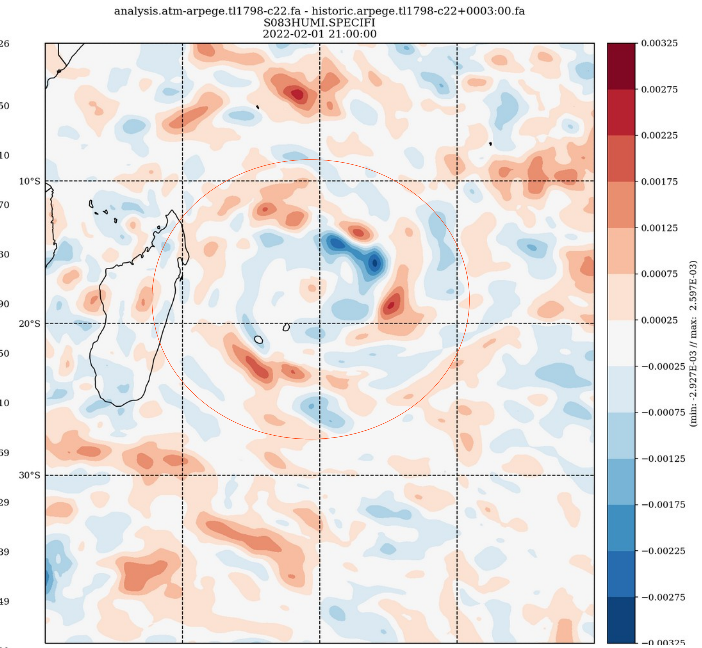
- **3D-hybrid covariances (B_0)** : 50% localised ensemble covs + 50% wavelet-filtered covs

$$B_0^h = \gamma^{e2} B_0^e + \gamma^{m2} B_0^m$$

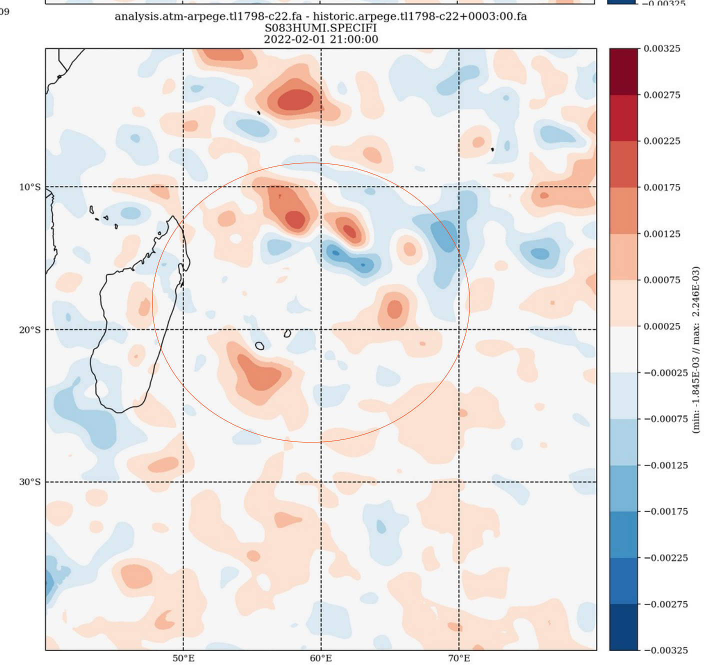
Tropical cyclone case (Batsirai)



Background field
of specific humidity
(850 hPa)

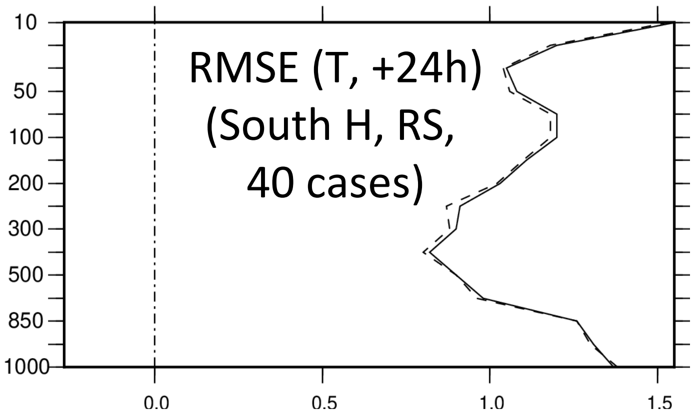


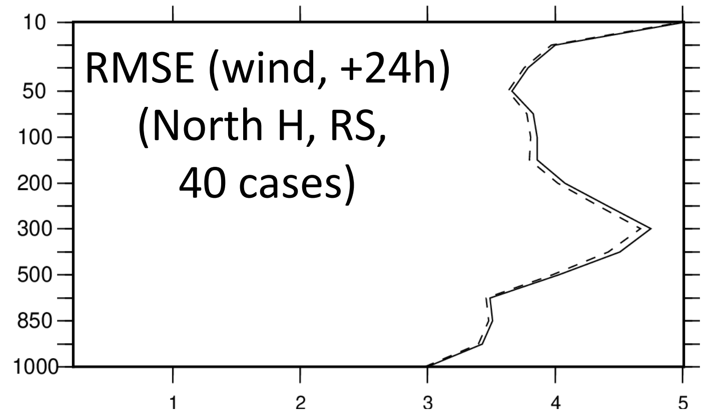
Increment with
localised covariances
(~ anisotropic)

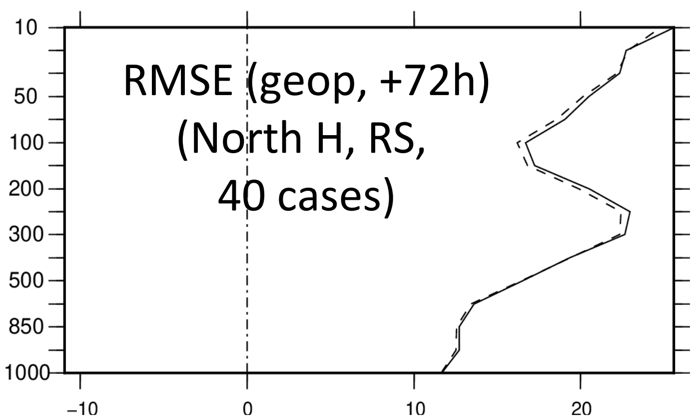


Increment with
wavelet covariances
(~ isotropic)

Impact of anisotropic covariances (3D-hybrid covs, in 4D-Var with OOPS)







North H (60 cases)	Réf. / Grille / Éch.	Radiosonde verification: 0H à 96H pas de 12H	IFS analysis verification: 0H à 102H pas de 6H
Geopotential	100hPa	▼ ▼ ▼ ▼ ▼ ▼ ▼ ▼ ⁞	▲▲▲▲▼▼▼▼▼▼▼⁞⁞=====
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———— 4D-Var with isotropic covs (wavelets)

- - - - - 4D-Var with anisotropic covs (3D-hybrid)

=> included in current E-suite at MF with OOPS.

4D extension of covariance hybridisation

Hybridisation of 3D covariances (at beginning of 6h DA window) :

$$\mathbf{B}_0^h = \gamma^{e2}\mathbf{B}_0^e + \gamma^{m2}\mathbf{B}_0^m$$

3D ens. covs + wavelet covs

Hybridisation of 4D covariances (throughout the 6h DA window) :

$$\underline{\mathbf{B}}^h = \gamma^{e2}\underline{\mathbf{B}}^e + \gamma^{m2}\,\underline{\mathbf{M}}\mathbf{B}_0^m\underline{\mathbf{M}}^T$$

4D ens. covs + linearly propagated covs (by tl/ad)
(implicit in 4D-Var, explicit here)
(can be computed at low resolution)

(Berre and Arbogast 2023, QJRMS)

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(can be computed at low resolution)

=> Intermediate scheme between « pure » 4DEnVar and 4D-Var ;

implementation is eased by the object-oriented framework (**OOPS**).

=> Facilitates **spatial resolution increases** of analysis increments

(ensemble information on physical processes that are difficult to simplify and linearise) ;

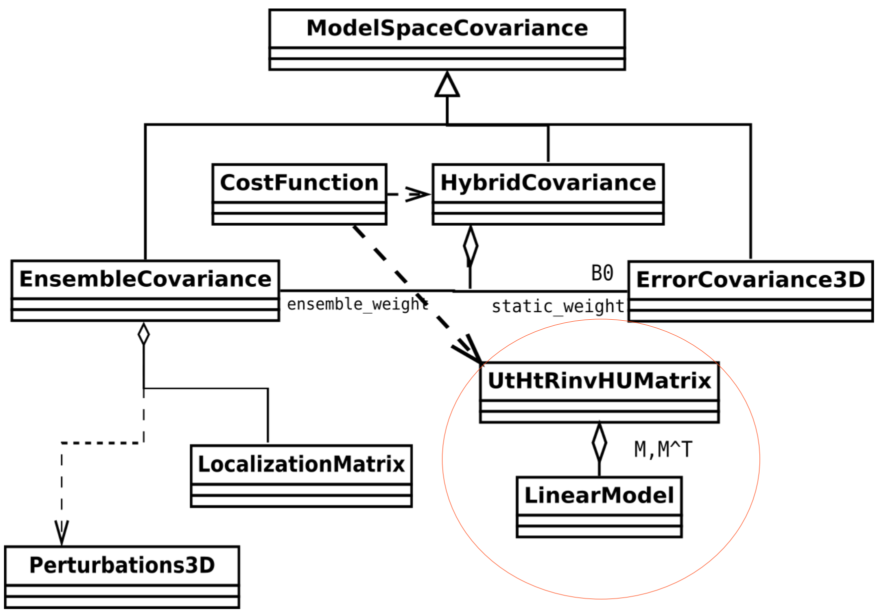
progressive transition from 4D-Var to 4DEnVar ; avoids «hybridisation with static covs».

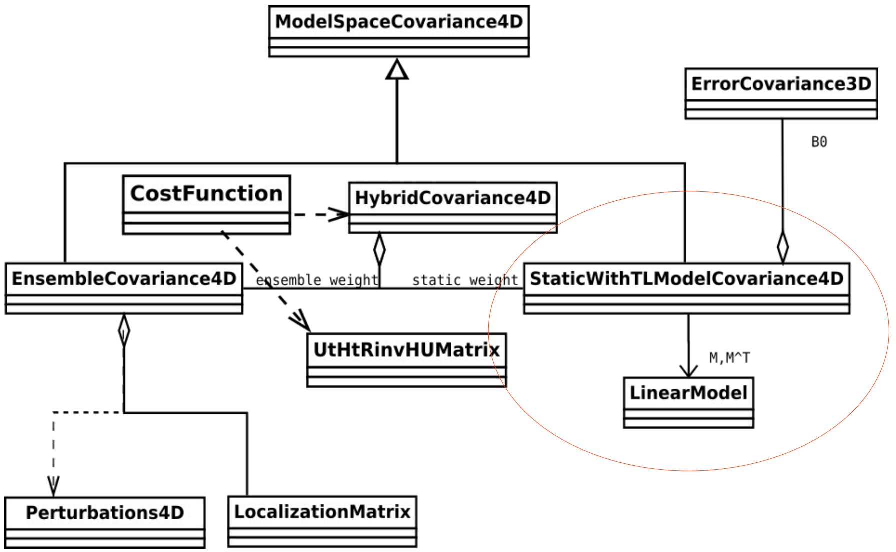
=> Facilitates **representation of model errors** :

synergy with model parameter perturbations in EDA and EPS.

(Berre and Arbogast 2023, QJRMS)

Flexibility of OOPS :
one can use M,M^T « in different ways »





4D-Var
(with 3D-hybrid covs) :
M is part of **H'**
(**H'** = **H** **M**)

4DEnVar
(with 4D-hybrid covs) :
M is part of **B**
(**M**B_0**M**^{T})

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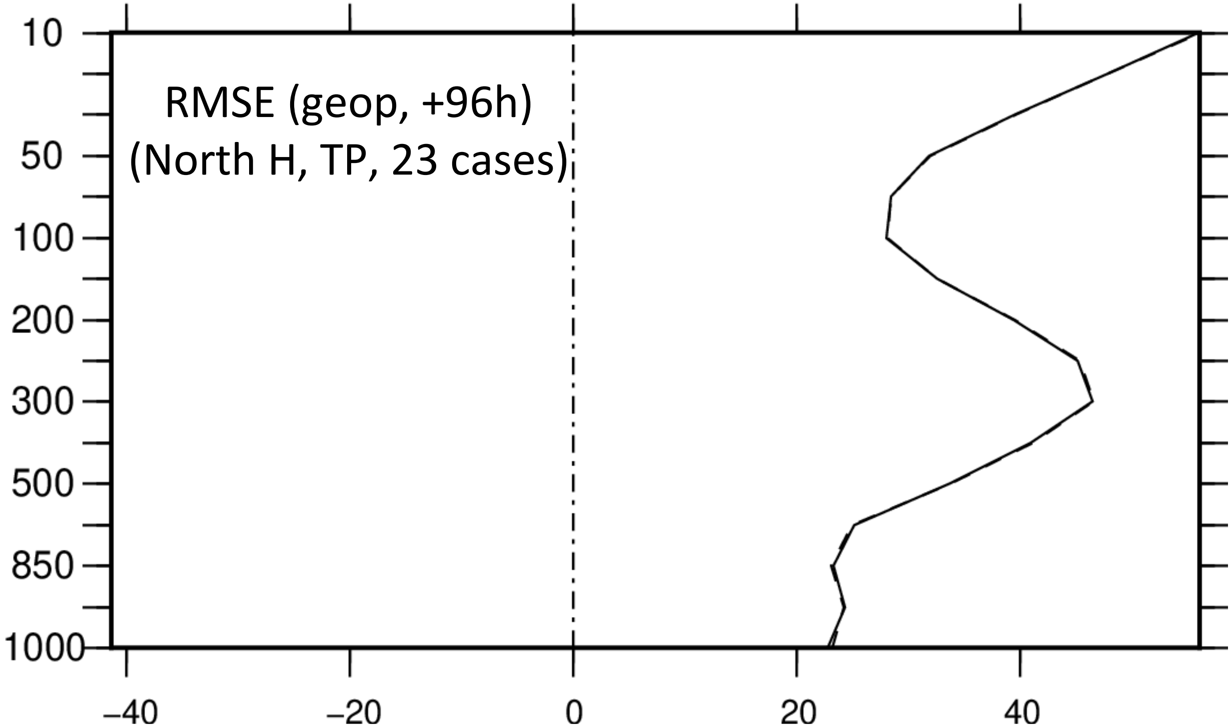
Set of experimental comparisons

Evaluation	Experiment		Reference	γ^{e2}	γ^{m2}
Equivalence between	4DEnVar with **linear covs only**	and	4D-Var	0	1
Impact of	4DEnVar with **4D-hybrid covs**	vs	4DEnVar with static covs	0.5	0.5
Impact of	**4DEnVar** with 4D-hybrid covs	vs	4D-Var with 3D-hybrid covs	0.5	0.5

Equivalence between 4D-Var and « 4DEnVar based on linear covs » ($\gamma^m = 1$ & $\gamma^e = 0$)



4D-Var with $\underline{\mathbf{H}}' = \underline{\mathbf{H}}\ \underline{\mathbf{M}}$

4DEnVar with $\underline{\mathbf{M}}\mathbf{B}_0\underline{\mathbf{M}}^T$ (γ^m=1 and γ^e=0)

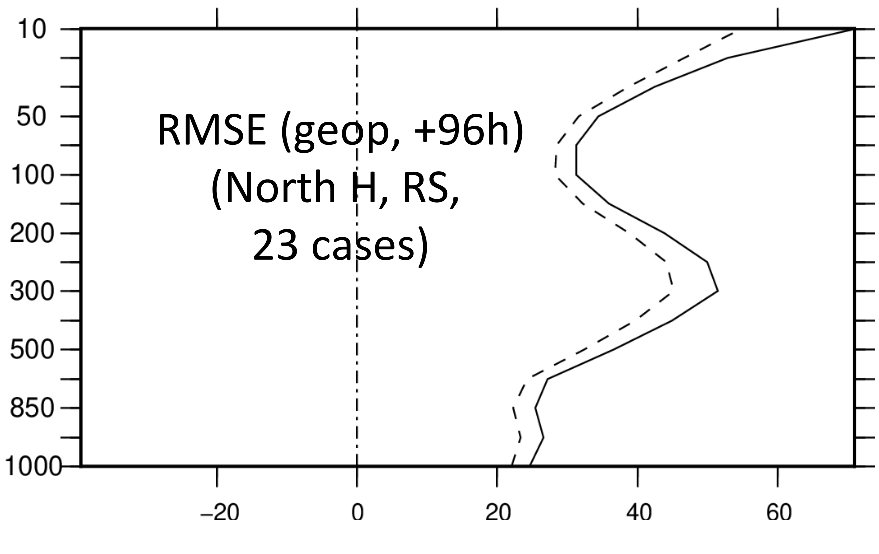
=> A good development basis for 4DEnVar, with same quality as 4D-Var.

It is equivalent to minimise :

- either (as in 4D-Var) the distance to a 3D background and to 4D observations, with the model $\underline{\mathbf{M}}$ being part of $\underline{\mathbf{H}}' = \underline{\mathbf{H}}\ \underline{\mathbf{M}}$;

- or (as in 4DEnVar) the distance to a 4D background and to 4D observations, with the model $\underline{\mathbf{M}}$ being part of $\underline{\mathbf{B}} = \underline{\mathbf{M}}\mathbf{B}_0\underline{\mathbf{M}}^T$

4DEnVar : 4D hybridisation vs static hybridisation



4DEnVar with static hybridisation

$$\underline{B}^h = \gamma^{e2}\underline{B}^e + \gamma^{m2}\,\underline{1}B_0^m\underline{1}^T$$

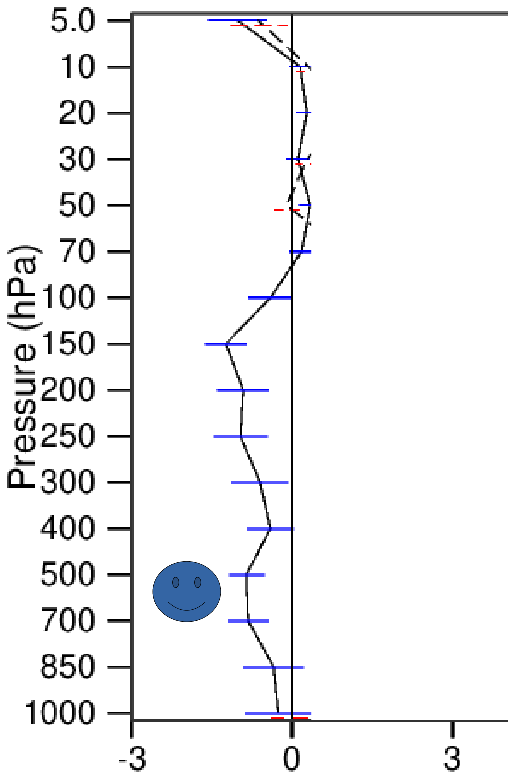
(γ^e=0.5 and γ^m=0.5)

4DEnVar with 4D hybridisation

$$\underline{B}^h = \gamma^{e2}\underline{B}^e + \gamma^{m2}\,\underline{M}B_0^m\underline{M}^T$$

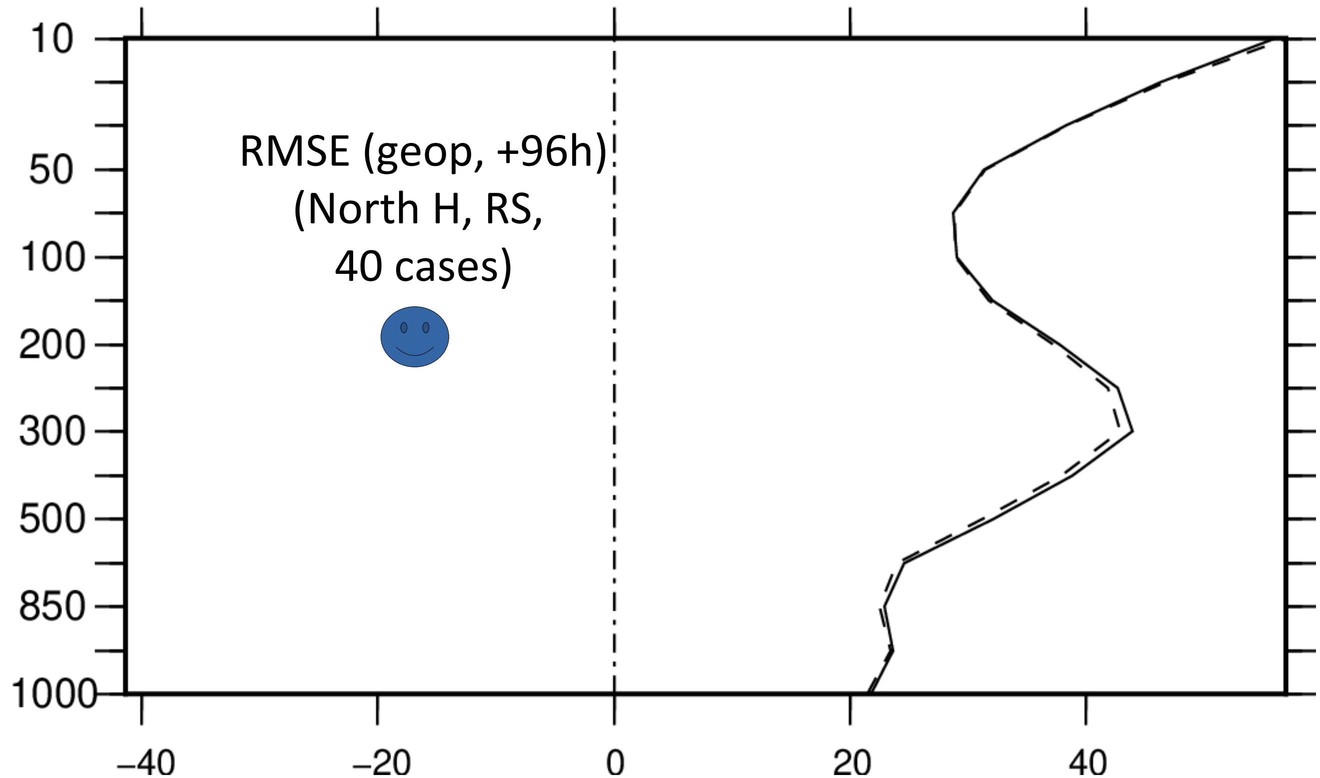
(γ^e=0.5 and γ^m=0.5)

Impact of « 4DEnVar with 4D-hybrid covs » against « 4D-Var with 3D-hybrid covs », over Northern Extratropics



%diff RMS(obs-guess)
(Temperature RS,
North H, 2 months)

(4DEnVar with 4D-hybrid covs
vs 4D-Var with 3D-hybrid covs)



4D-Var with 3D-hybrid covs
(γ^m=0.5 and γ^e=0.5)

4DEnVar with 4D-hybrid covs
(γ^m=0.5 and γ^e=0.5)

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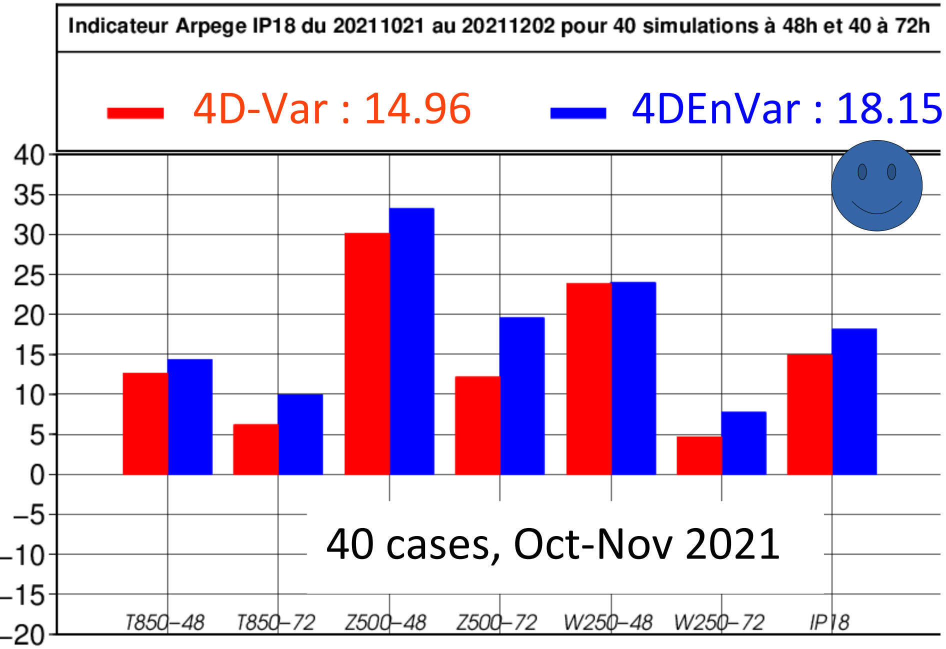
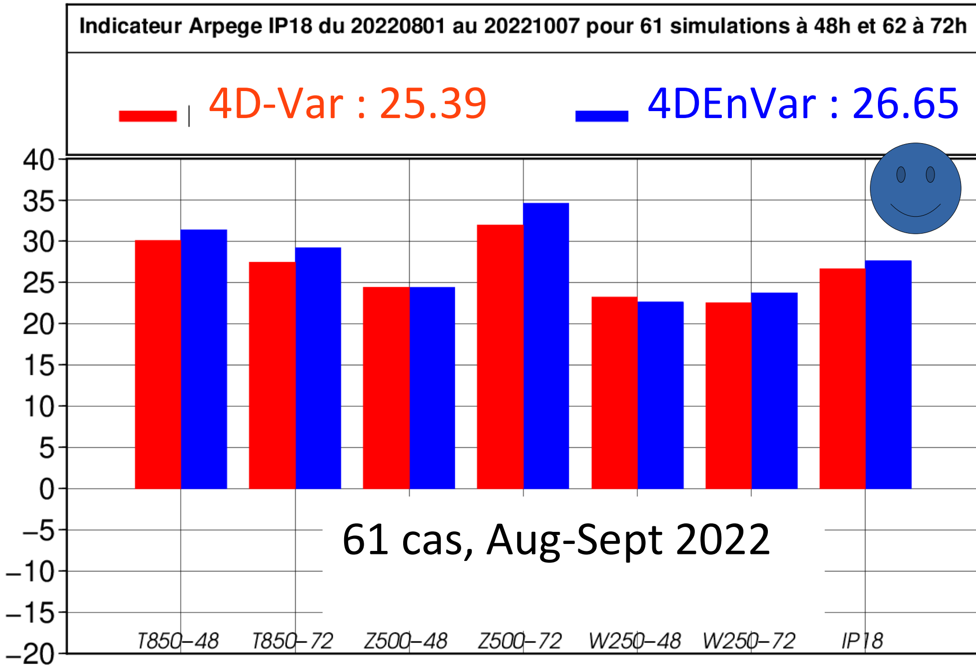
Impact of « 4DEnVar with 4D-hybrid covs » against « 4D-Var with 3D-hybrid covs », over Europe





The more accurate the forecasts are (in terms of RMSE), the larger the forecast quality index (IP18) is.

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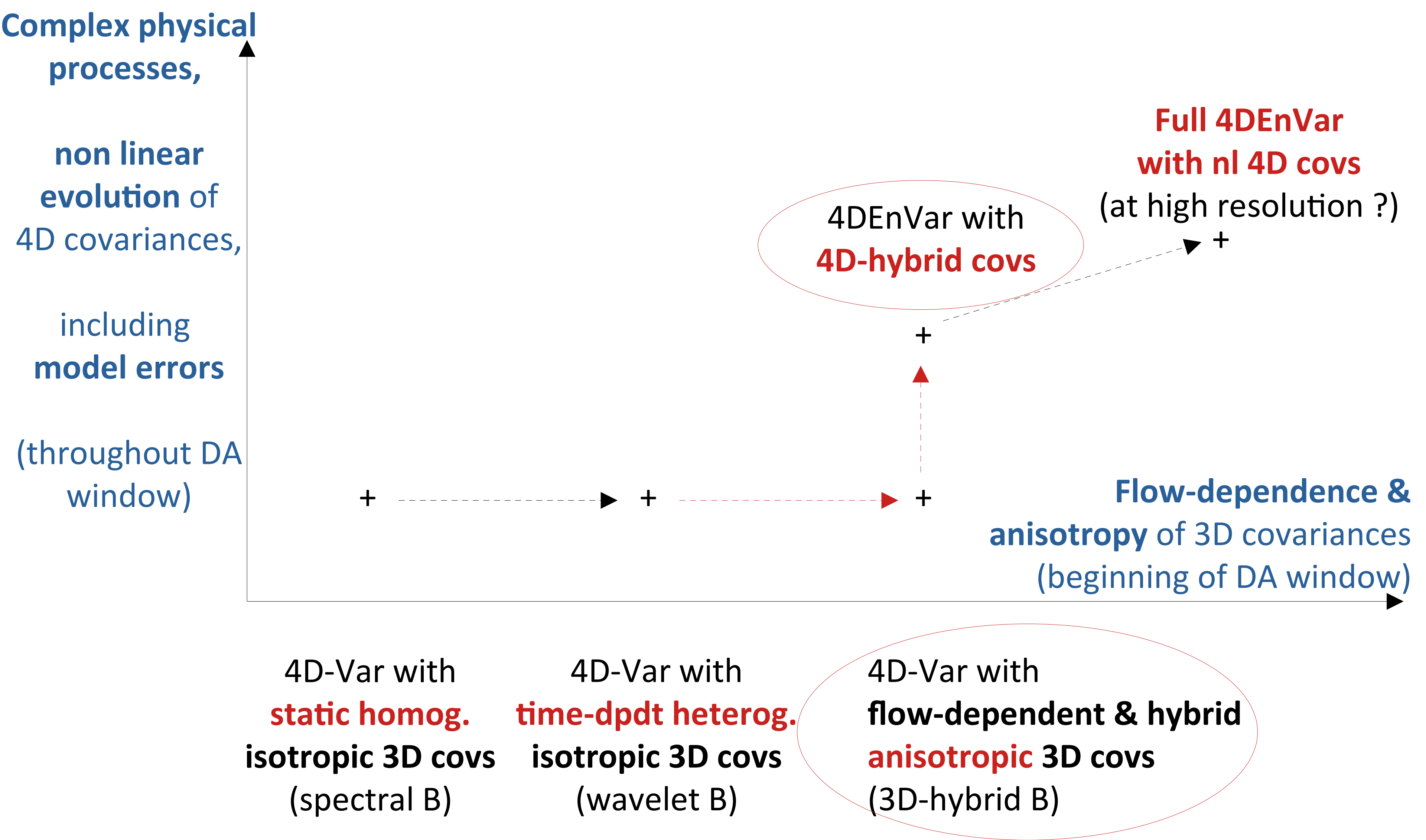
Progressive increase of the use of ensembles in global Var DA at MF



Conclusions

- Implementation of **3D-hybrid covariances** in ARPEGE (global DA) with OOPS : **flow-dependent anisotropies**, with positive impacts on forecast scores ; included in current real-time E-suite at Météo-France with OOPS.

- **4DEnVar** formulation and experiments with **4D-hybrid covariances**, in OOPS : facilitates representation of **physical processes** at high resolution and of **model errors** within DA window ; competitive with 4D-Var.

- Further experiments with **model parameter perturbations** in EDA. Connections with future **increases of DA spatial resolution** : either still keep some linear propagation, computed at low resolution, or move to full 4DEnVar at high resolution (e.g. see J.-F. Caron's talk).

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Thanks for listening !