Observation-space localization methods for the maximum likelihood ensemble filter

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Which localization is suitable for nonlinear optimization?

Abstract

We compare localization schemes in the maximum likelihood ensemble filter[1] (MLEF): two observation-space localizations (OL) and a combination of OL in the horizontal and state-space localization in the vertical (VSL). For nonlinear observations in the simplified atmospheric general circulation model, the OL with global optimization works better than that without optimization or with local optimization owing to a larger cost reduction. Moreover, the OL with global optimization shows comparable performance to VSL for nonlocal observations, suggesting that the global optimization can accommodate nonlinear and nonlocal observations such as satellites.

MLEF

Cost function with control variable transformation :

$$\mathbf{x} = \overline{\mathbf{x}^f} + \mathbf{P}_f^{\frac{1}{2}}\mathbf{w},\ [\mathbf{P}_f^{\frac{1}{2}}]_i = \mathbf{p}_i^f = (K-1)^{-\frac{1}{2}}(\mathbf{x}_i^f - \overline{\mathbf{x}^f})$$

$$2J = \mathbf{w}^T\mathbf{w} + [\mathbf{y} - H(\mathbf{x})]^T\mathbf{R}^{-1}[\mathbf{y} - H(\mathbf{x})]$$

Analyses are obtained from the optimization of J.

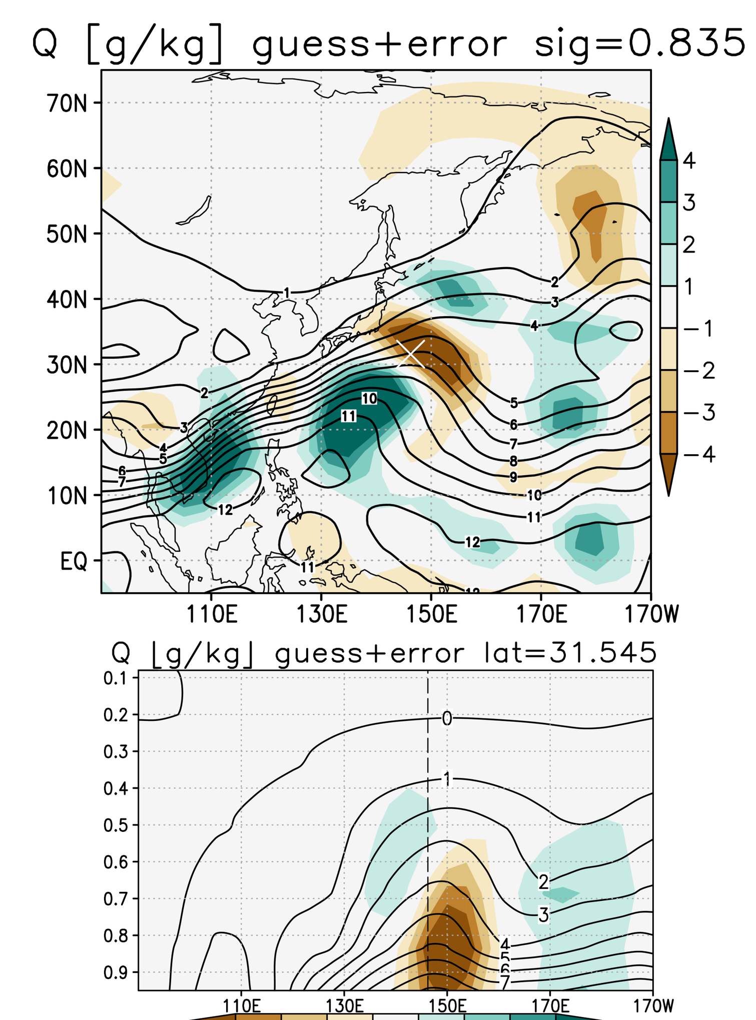
$$\overline{\mathbf{x}^a} = \overline{\mathbf{x}^f} + \mathbf{P}_f^{\frac{1}{2}}\mathbf{w}^{opt},\ \mathbf{P}_a^{\frac{1}{2}} = \mathbf{P}_f^{\frac{1}{2}}[\nabla^2 J(\mathbf{w}^{opt})]^{-\frac{1}{2}}$$

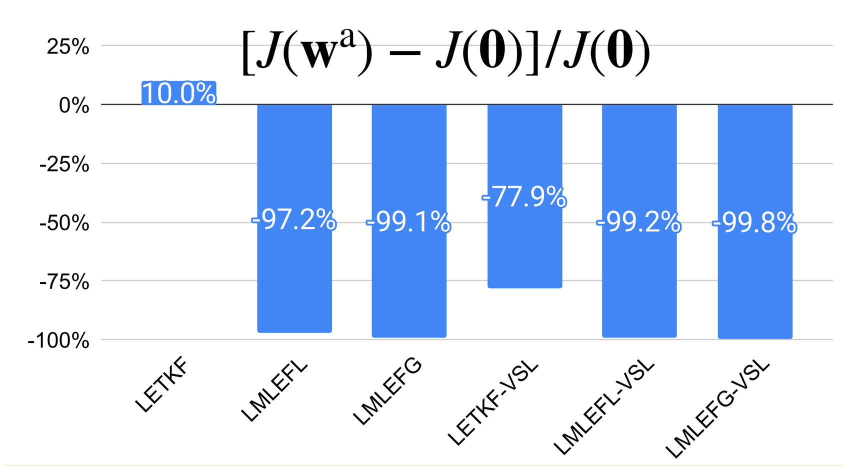
If H is linear, the analysis is equivalent to ETKF.

Single observation assimilation

Model: SPEEDY[7] T30L7
Ensemble size: 20
Localization scale:
900 km (H), $0.1 \ln p$ (V)
Observation (146.3E, 31.5N):

$$\int_{\sigma=0.95}^{\sigma=0.51} \ln Q \, d\sigma$$



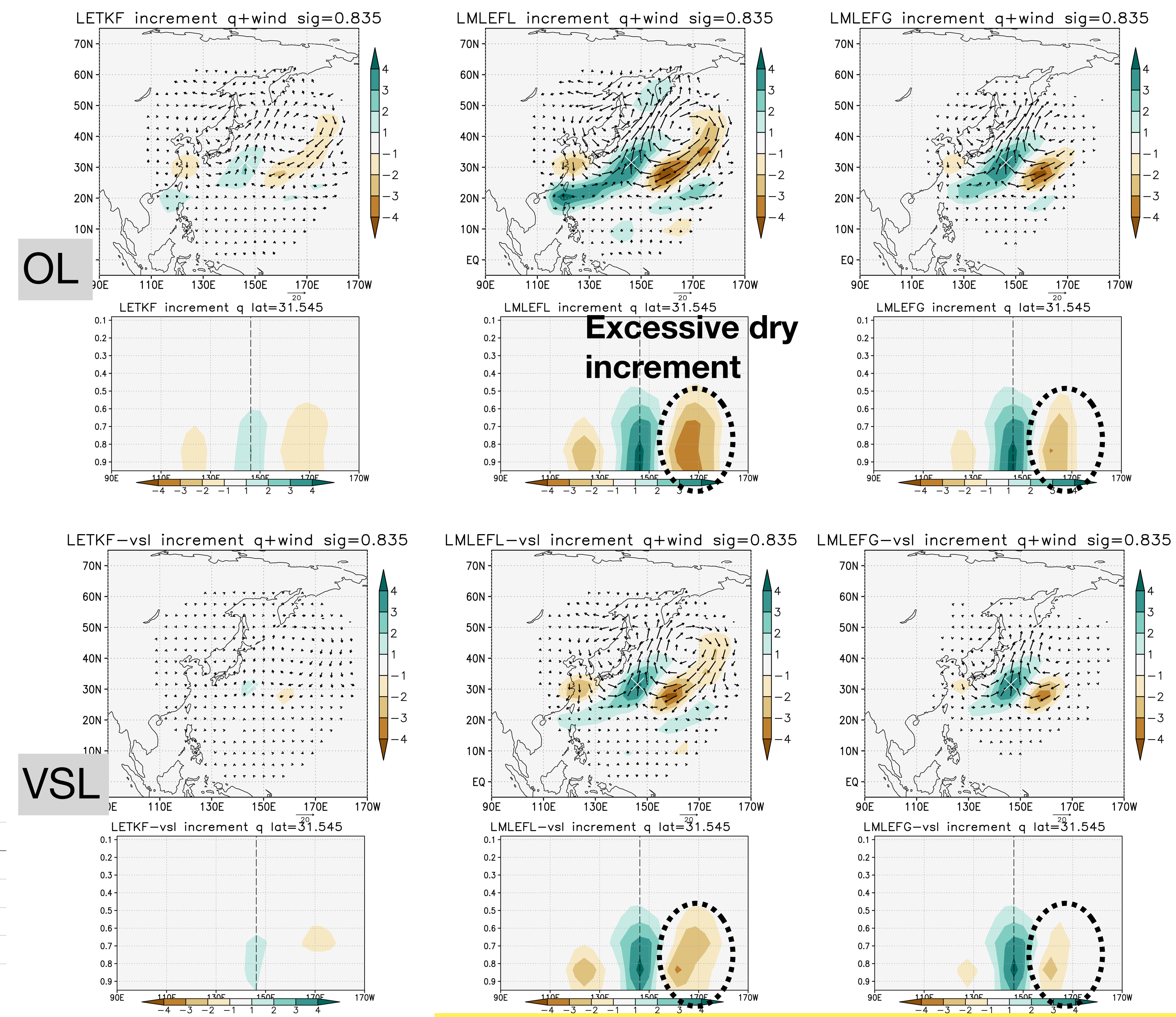


More effective with VSL

No optimization: little modification

Local optimization: broad impact

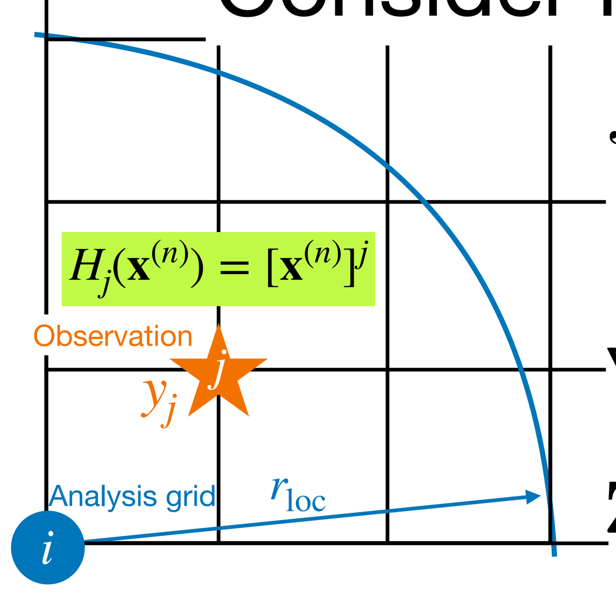
Global optimization: severe localization



VSL and global OL ameliorates negative dry increment.

Optimization with OL

Consider local cost at n-th iteration.



$$J_i^{(n)} = \frac{1}{2}\mathbf{w}_i^{(n)\mathrm{T}}\mathbf{w}_i^{(n)} + \frac{1}{2}[\mathbf{y} - H(\mathbf{x}^{(n)})]^{\mathrm{T}}\mathbf{R}_i^{-1}[\mathbf{y} - H(\mathbf{x}^{(n)})]$$

$$\nabla_{\mathbf{w}} J_i^{(n)} = \mathbf{w}_i^{(n)} - \mathbf{Z}_i^{(n)\mathrm{T}}\mathbf{R}_i^{-1/2}[\mathbf{y} - H(\mathbf{x}^{(n)})]$$

$$\mathbf{Z}_i^{(n)} = \mathbf{R}_i^{-1/2}[H(\mathbf{x}^{(n)} + \mathbf{X}_f) - H(\mathbf{x}^{(n)})]$$

$\mathbf{w}$ around y_j are required to update $H_j(\mathbf{x})$, which prevents independent analysis of OL.

Global optimization (LMLEFG)

Update $\mathbf{x}^{(n)}$ globally in each iteration[2,3]. This is equivalent to minimizing $\Sigma_i\ J_i$.

Local optimization (LMLEFL)

Update $[\mathbf{x}^{(n)}]^j$ by $\mathbf{w}_i^{(n)}$ assuming locally constant weights[4]. J_i are minimized independently.

Vertical state-space localization

A square root of vertically localized $\mathbf{P}_\mathrm{f}$ is constructed by modulated ensemble[5,6]:

$$\mathbf{P}_{\mathrm{loc}}^{\frac{1}{2}} = [\cdots, \mathrm{Diag}(\mathbf{p}_i^{\mathrm{f}})\mathbf{L}_{\mathrm{v}}^{\frac{1}{2}}, \cdots]$$

The augmented ensemble size becomes $K \times N_\mathrm{v}$ (number of vertical layers). The K analysis ensemble for each vertical column is obtained from the modified gain formulation[6].

References

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[3] Bocquet, M. 2016: *QJRMS*, 142, 1075–1089.
[4] Yang, S-C. et al. 2009: *QJRMS*, 135, 251–262.
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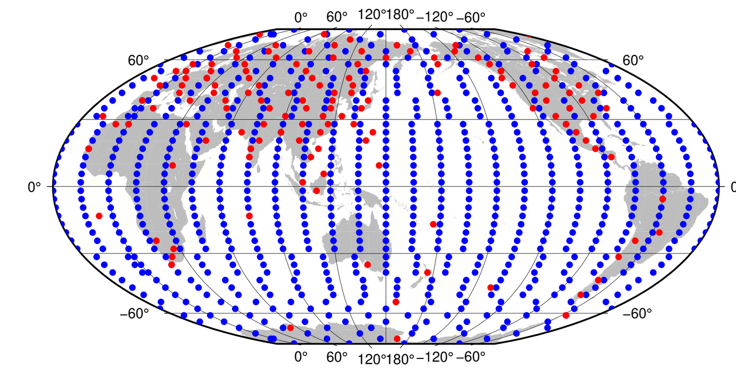
Cycling experiments

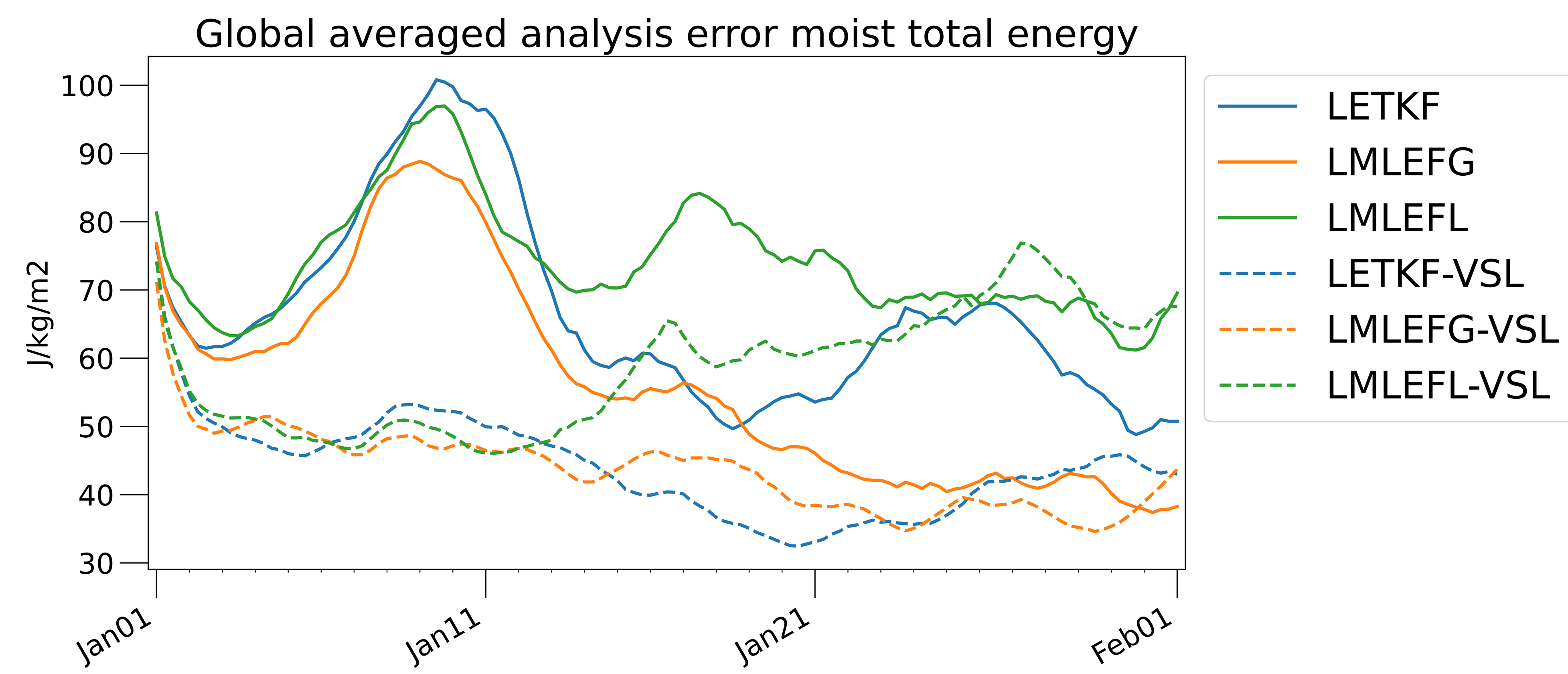
Inflation: adaptive multiplicative[8]
Experimental period: January
Cycling interval: 6 hours



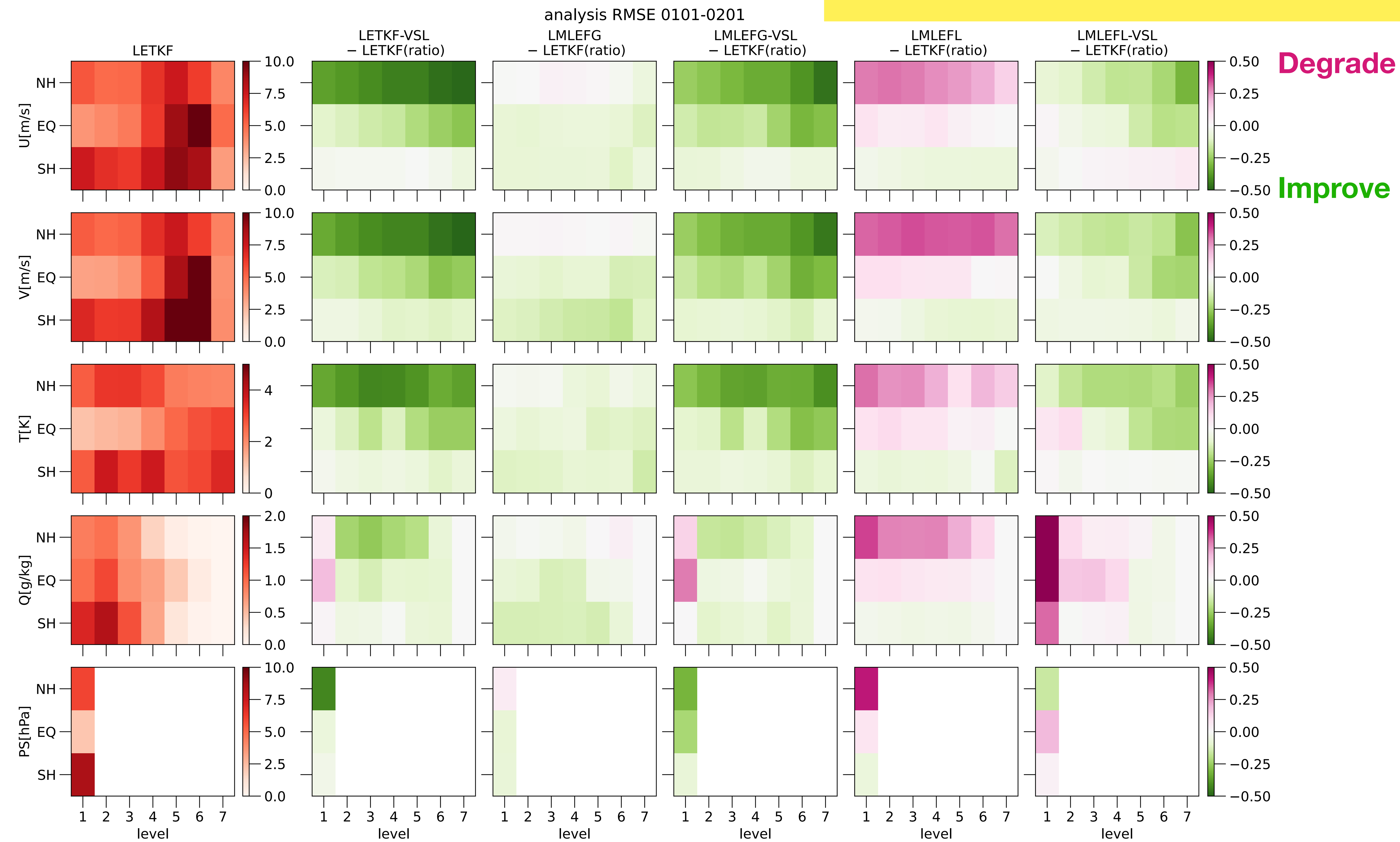
Observations

Linear (139): U, V, T (7 levels), Q (4 levels), Ps

Nonlinear (748) $\int \ln Q \, d\sigma$



- VSL shows better performance than OL in the early cycles.
- The global optimization with OL reaches comparable quality to VSL in later cycles.
- The local optimization may deteriorate the analysis in the dense observed area.



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