

GHOSH v1.0.0: a novel Gauss-Hermite High-Order Sampling Hybrid filter for computationally efficient data assimilation in geosciences

Simone Spada¹ (sspada@ogs.it), Anna Teruzzi¹, Stefano Maset², Stefano Salon¹, Cosimo Solidoro¹, Gianpiero Cossarini¹

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¹National Institute of Oceanography and Applied Geophysics - OGS, Trieste, Italy

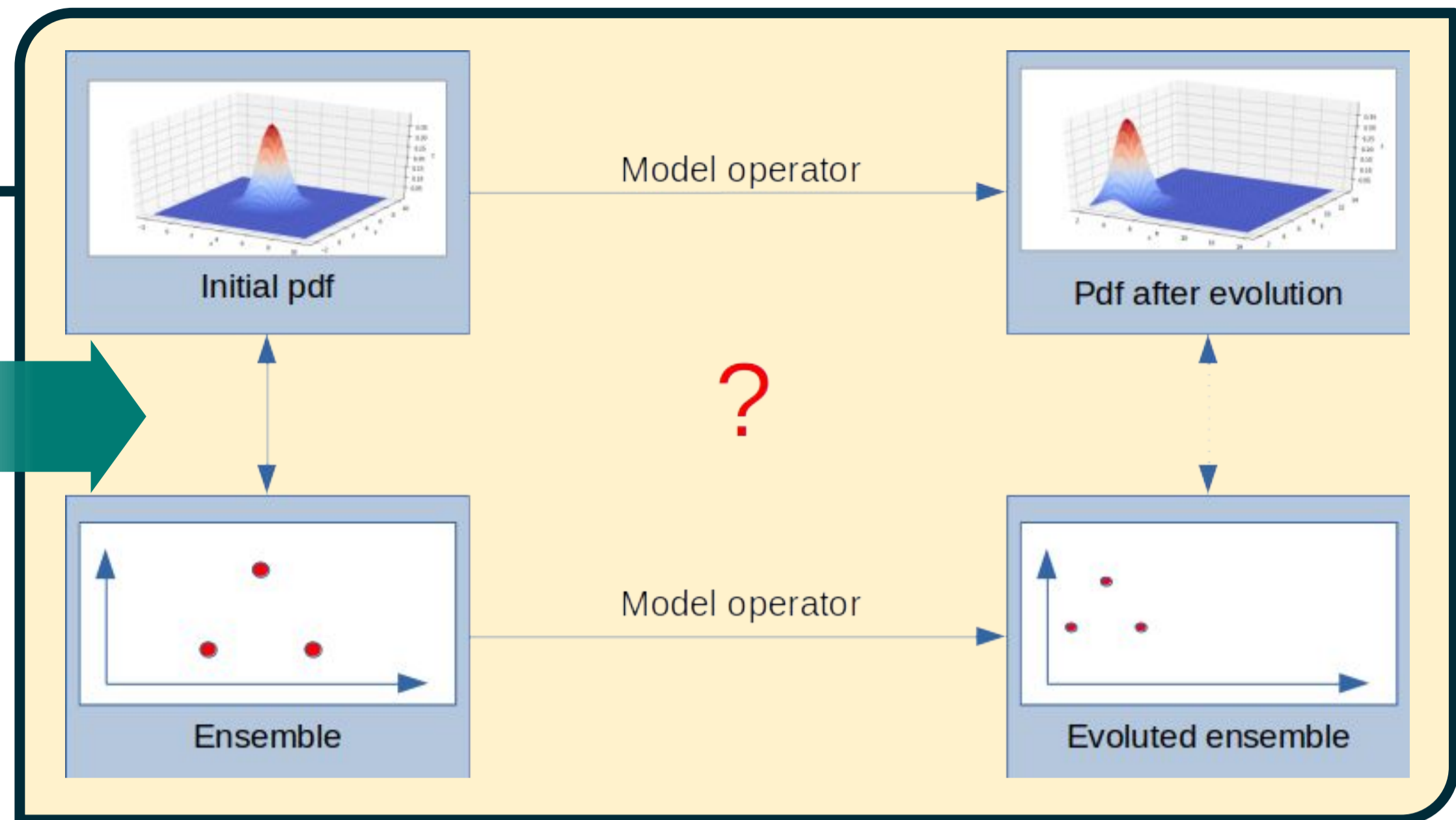
²Universita' degli Studi di Trieste, Dipartimento di Matematica e Geologia, Trieste, Italy

The GHOSH sampling method

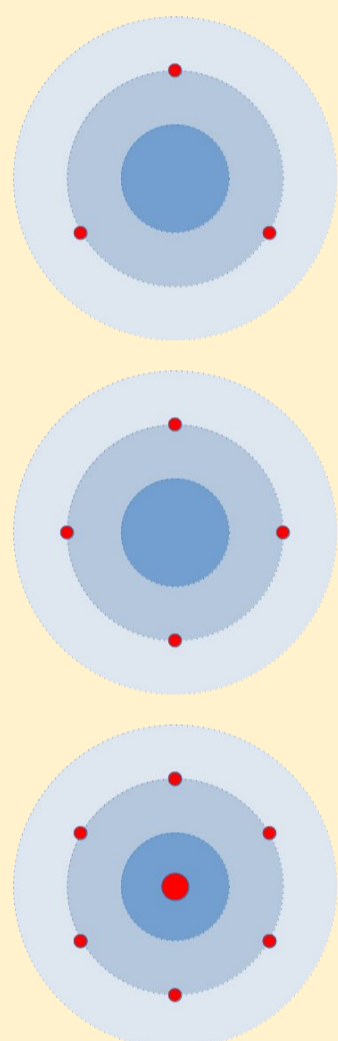
Three facts you want to know about ensembles:

1. An ensemble is a representation of the probability density function (pdf) of the uncertainty of the state of the system
2. Evolving the ensemble is not a good proxy of the evolution of the uncertainty pdf, i.e., **the diagram is not commutative!**
3. The ensemble mean after the evolution is affected by an error that can be reduced by increasing the **order** of the sampling method.

A sampling has **order** h if the ensemble mean is exact after evolving the pdf with any polynomial model operator of order not bigger than h



Sampling and order examples



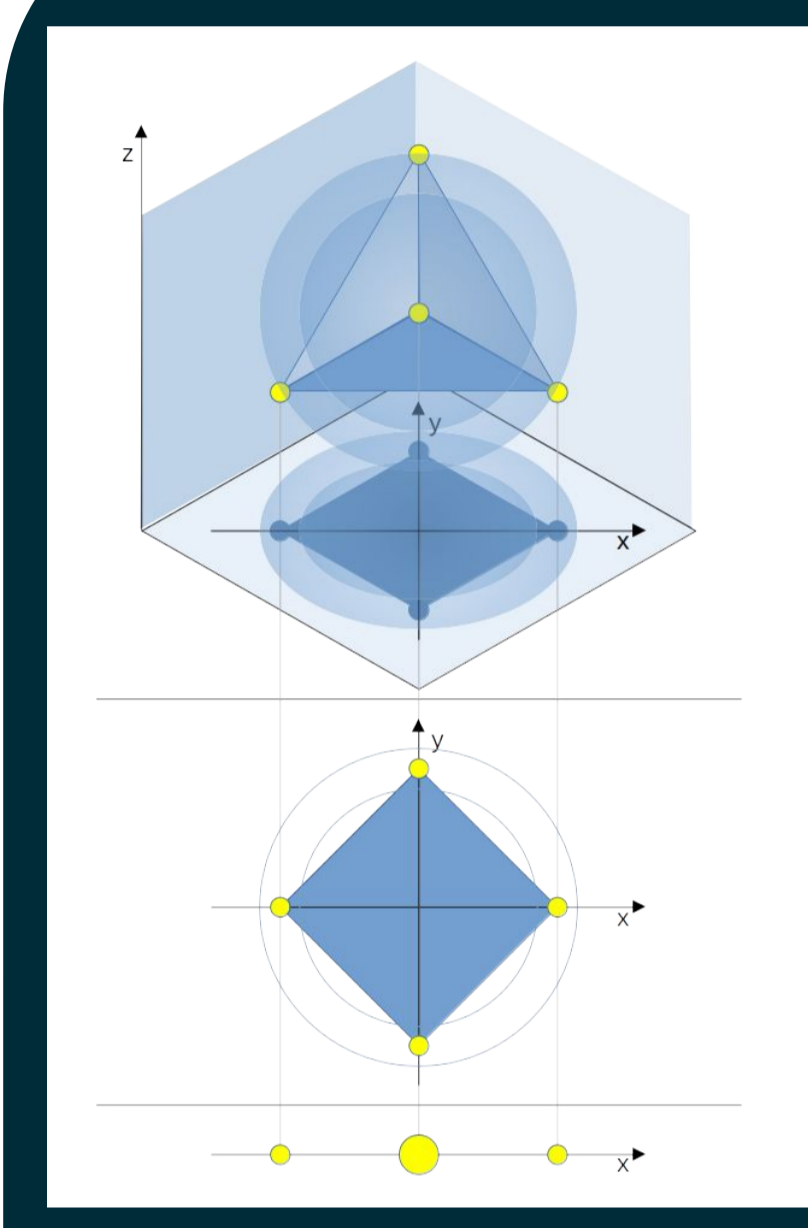
3 ensemble members:
2nd order sampling
This is the most used sampling in square root / deterministic filters

4 ensemble members:
3rd order sampling

7 weighted ensemble members:
5th order sampling

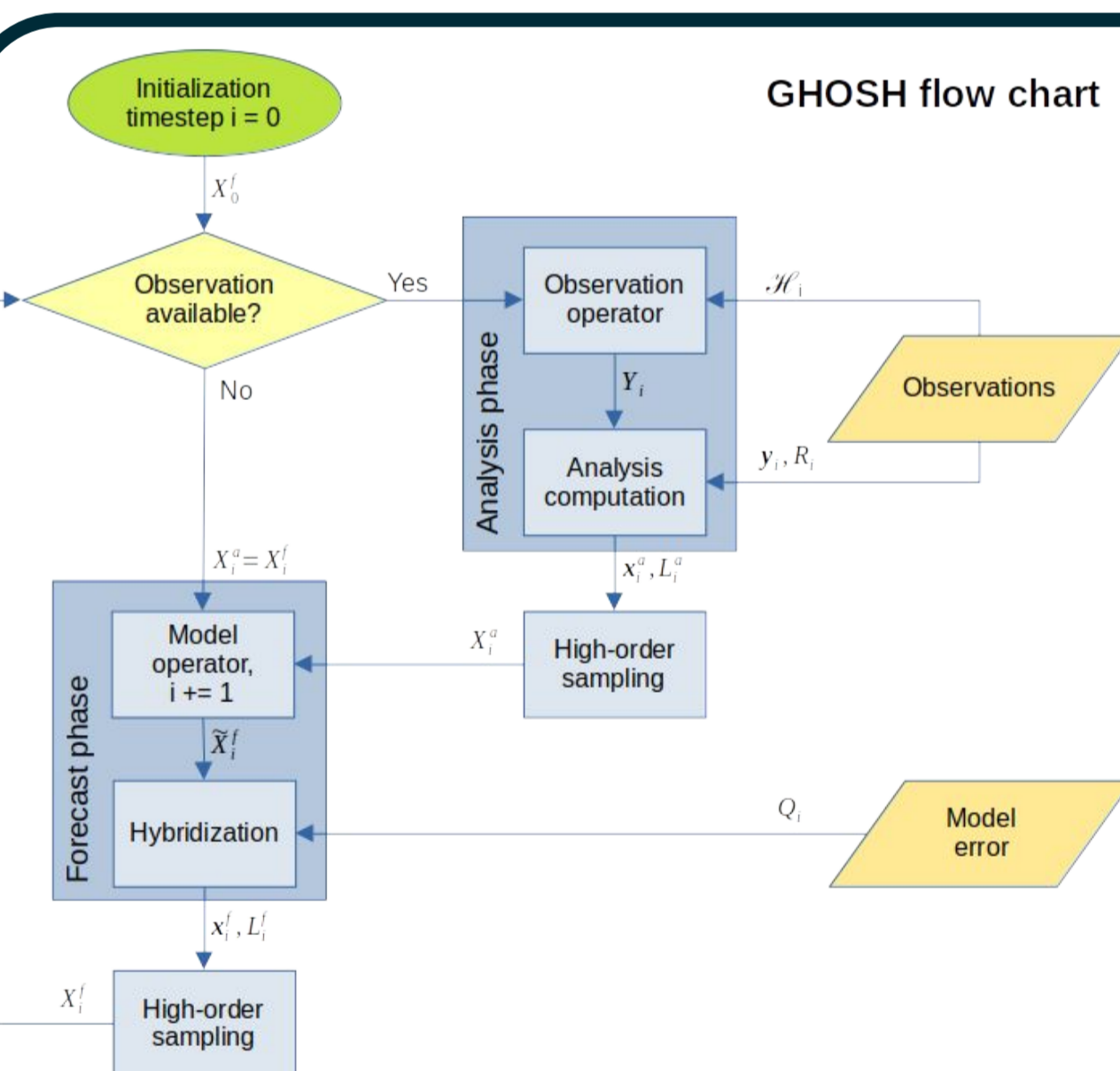
Improved precision
is obtained by
rising order
which requires
more members
but implies
higher computational cost

A new SOLUTION



4 members in 3D
(2nd order approximation)
that project in
4 members in 2D
(3rd order approximation)
that project in
3 weighted members in 1D
(5th order approximation)

Improved precision
by
rising order
in the most
relevant PCA
components
NO more members
NO higher computational cost

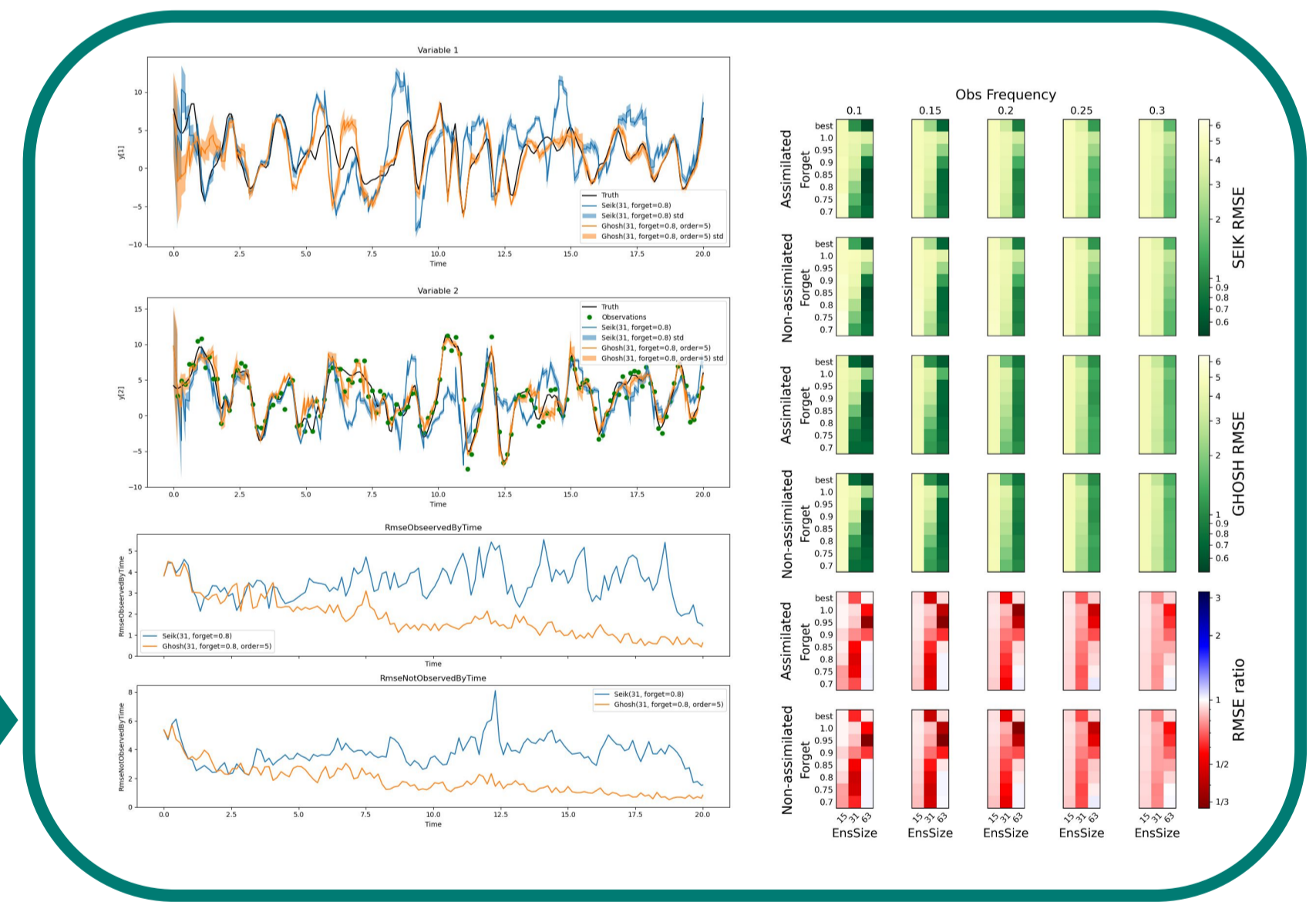


The GHOSH filter

- Features:**
- Two **high-order** resamplings
 - **Weighted ensemble**
 - Hybrid covariance
 - Improved analysis equations
 - **Same computational cost** as a common square root filter (e.g., SEIK, ETKF)

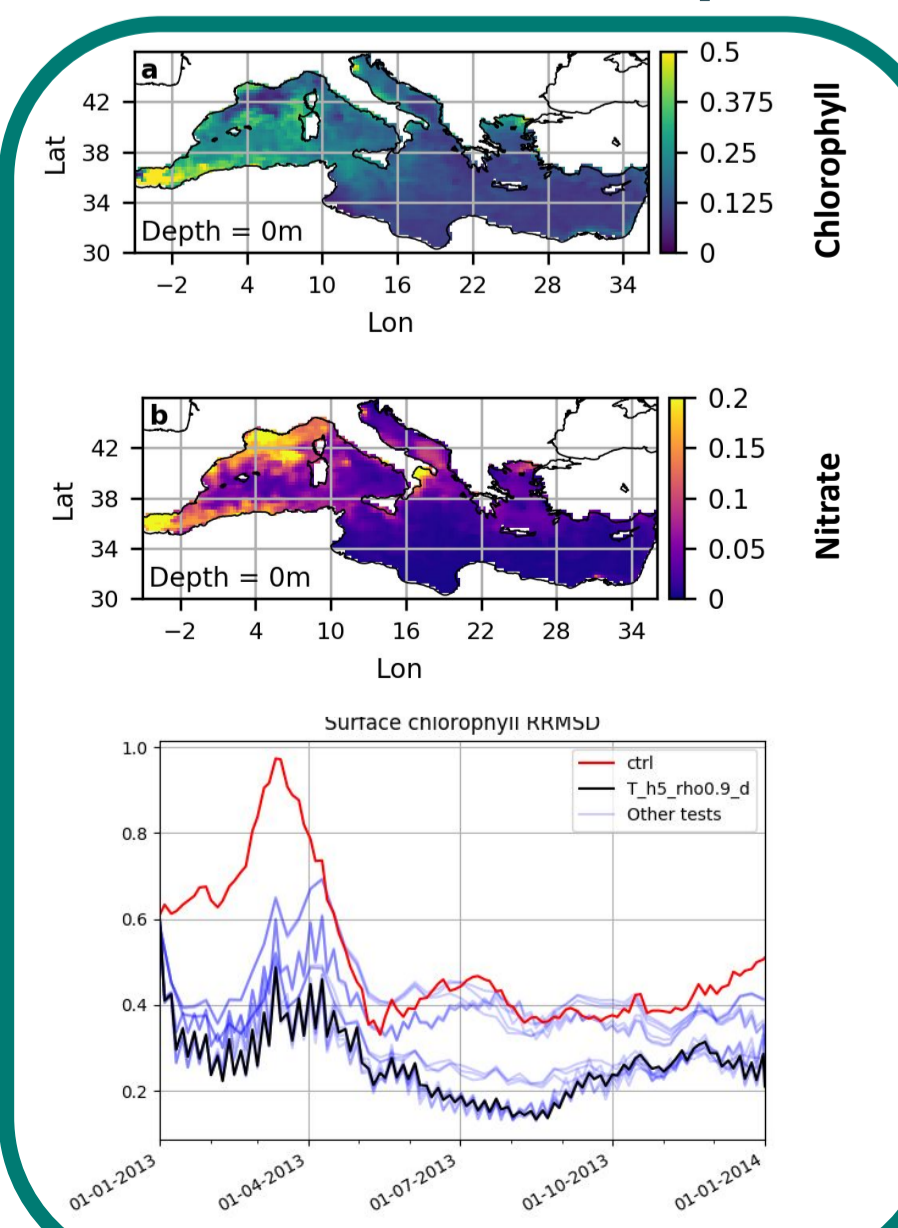
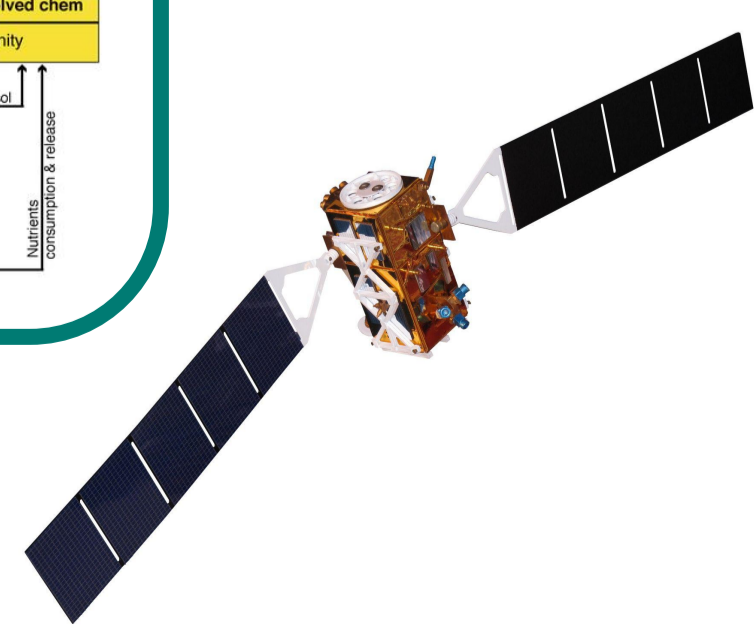
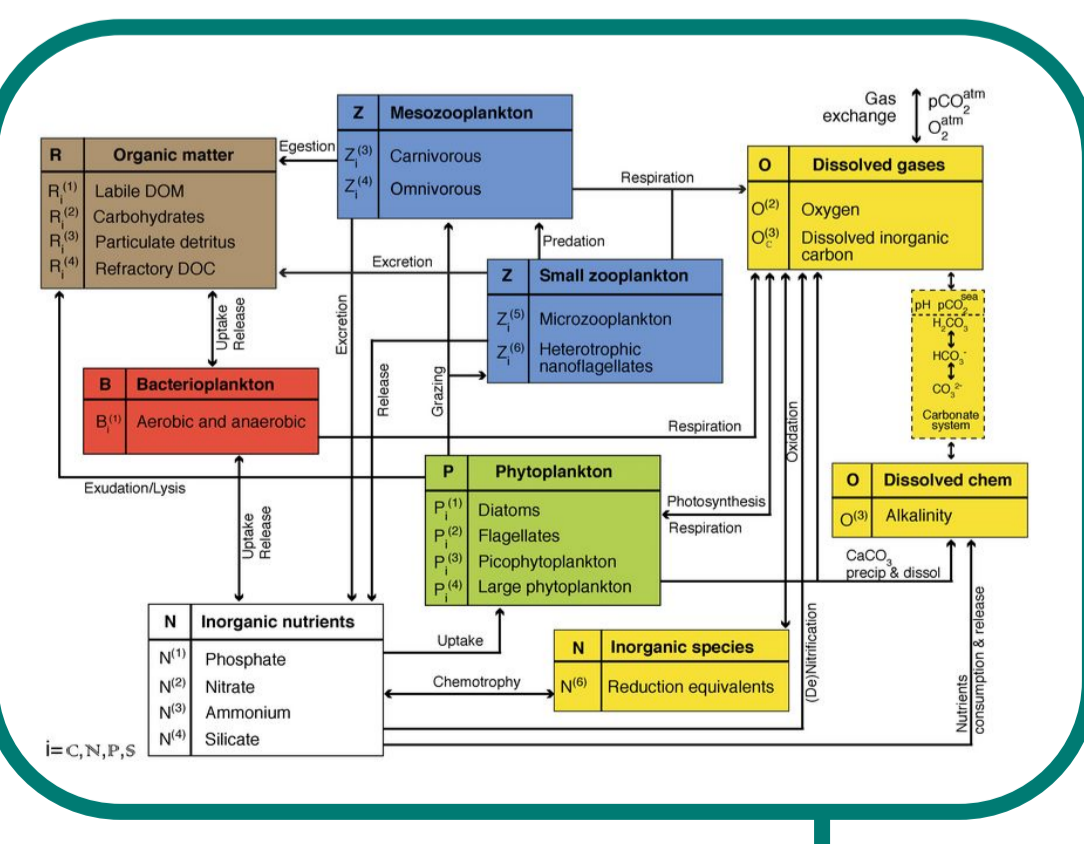
Twin experiment (GHOSH vs SEIK):

- Lorenz96 model (56000 tests)
- **GHOSH up to 3 times better** than SEIK
- **GHOSH improves stability**



3D application in the Mediterranean Sea: Copernicus Marine Service transport-biogeochemical coupled model

OGSTM-BFM model + Satellite chlorophyll observations



RESULTS

GHOSH parallel realistic implementation:

- Feasible
- Improves RMSD
- Does not degrade non-assimilated variables
- Over 18 tests, the order resulted highly correlated with improvements in assimilation skill
- Same computational cost as other ensemble method