Transgressions in the relative Weil algebra

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Let G be a Lie group and $\mathfrak{g}$ be its Lie algebra.

The transgression map between invariants in the symmetric algebra $S(\mathfrak{g}^*)$ of the dual space $\mathfrak{g}^*$ and invariants in the exterior algebra $\Lambda(\mathfrak{g}^*)$ of $\mathfrak{g}^*$ was define by H. Cartan in 1950.

Later it appear in the work of Chern and Simons on the theory of G-principal bundles.

The transgression map can be constructed using cohomological properties of the Weil algebra $W(\mathfrak{g})$ of $\mathfrak{g}$.

The Weil algebra $W(\mathfrak{g}) = S(\mathfrak{g}^*) \otimes \Lambda(\mathfrak{g}^*)$ is a differential graded algebra introduced by A. Weil as an algebraic model for differential forms on the classifying bundle of G.

We generalise this construction to the relative case of symmetric spaces.

Let G be a real reductive Lie group and $\mathfrak{g}$ be the complexification of its Lie algebra.

The Cartan involution on G induces the Cartan decomposition $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ of $\mathfrak{g}$. The corresponding relative Weil algebra is $W(\mathfrak{g}, \mathfrak{k}) = (S(\mathfrak{g}) \otimes \Lambda(\mathfrak{p}))^{\mathfrak{k}}$. We define the transgression map between $\mathfrak{g}$-invariants in $S(\mathfrak{g})$ and $\mathfrak{k}$-invariants in $\Lambda(\mathfrak{p})$ and study its properties.

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