NonPUB23

Nonlocal and Nonlinear PDEs at the University of Bologna

June 8-9, 2023

WORKSHOP BOOKLET

Abstracts

Geometrical estimates for the first eigenvalue of linear operators: the fractional Dirichlet-Laplacian and the Dirichlet-bilaplacian

Francesca Bianchi
Università degli Studi di Parma

June 9, 10:15

In this talk we discuss the existence of lower bounds in terms of the *inradius* of the set for the first eigenvalue λ_1^s of the fractional Dirichlet–Laplacian. We show that some topological constraints are needed: in particular, we prove such an estimate for the class of planar sets with a fixed number k of "holes" by exploiting the relation between the fractional capacity of a compact set and λ_1^s. We also discuss the optimality (in some sense) of its dependence with respect to the parameters s and k.

We finally arise this method of proof in different settings: in particular we show the existence of a lower bound in terms of the *inradius* for the first eigenvalue Λ_1 of the Dirichlet–bilaplacian in dimension $N = 2, 3$.

Some of the results presented are obtained in collaboration with Lorenzo Brasco (University of Ferrara).

Anisotropic p-Laplacean equations: the pursuit of a comprehensive theory of regularity

Simone Ciani
Alma Mater Studiorum Università di Bologna

June 8, 14:00

In this talk we will introduce an equation in divergence form whose principal part is the Euler-Lagrange equation of the en-

ergy integral

$$\mathcal{F}(u) = \sum_{i=1}^{N} \frac{1}{p_i} \int_{\Omega} \left|\partial_i u\right|^{p_i} \, dx, \qquad \Omega \subset\subset \mathbb{R}^N,$$

known to be the prototype of orthotropic non-standard growth functionals $\mathcal{F}$ in the Calculus of Variations. We will focus on the similarities and differences of the theory of regularity in comparison with the p-Laplacean one, and describe a problem which is still active and widely open after more than fifty years.

The lack of homogeneity calls for a new approach to a wider theory of regularity that encompasses the minimizers of $\mathcal{F}$: we will present some possible new methods, motivated by recent Harnack estimates and adapted from the theory of parabolic differential equations.

Nonlocal capillarity problems with anisotropic kernels

Alessandra De Luca
Università Ca' Foscari Venezia

June 9, 11:30

Classical capillarity theory is based on the study of volume-constrained critical points and local/global minimizers of the Gauss free energy of a liquid droplet occupying a region inside a container. In 2016 F. Maggi and E. Valdinoci introduced a family of nonlocal capillarity models where surface tension energies are replaced by fractional-type interaction energies. I will discuss about a nonlocal capillarity problem involving interaction kernels which are possibly anisotropic and have different homogeneity in order to take into account the possibility that the liquid/air interaction and the liquid/surface one are different.

Schauder estimates up to the boundary on H-type groups: an approach via the double layer potential

Gianmarco Giovannardi
Università degli Studi di Firenze

🕓 June 8, 16:45

We will show how to obtain the Schauder estimates at the boundary away from the characteristic points for the Dirichlet problem by means of the double layer potential in a Heisenberg-type group $\mathbb{G}$. Despite its singularity we manage to invert the double layer potential restricted to the boundary thanks to a reflection technique for an approximate operator in $\mathbb{G}$. This is the first instance where a reflection-type argument appears to be useful in the sub-Riemannian setting. This is a joint work with G. Citti and Y. Sire.

L^p contraction estimates for parabolic equations via the nonlinear adjoint method

Alessandro Goffi
Università degli Studi di Padova

🕓 June 8, 16:00

In this talk I will report on some works in progress concerning stability and contraction estimates in Lebesgue spaces, along with uniqueness properties, for linear and nonlinear time-dependent PDEs with local and/or nonlocal diffusion. I will first discuss a general strategy to obtain L^1 contraction estimates for nonlinear diffusion equations of generalized porous medium type. Then, I will outline some possible applications to other advection and convection-diffusion models, together with Kruzkov type L^1 stability estimates for solutions of conservation laws. Finally, I will focus on contraction and stability estimates in L^p spaces for first- and second-order Hamilton-Jacobi equations along with the rate of convergence of the vanishing viscosity

process. The latter results improve (with respect to the norm involved and the dependence on the constants) upon a rate of convergence obtained by M.G. Crandall, P.-L. Lions, and P.-L. Lions through viscosity solutions and probabilistic methods, and a more recent one by C.-T. Lin and E. Tadmor in L^1. This analysis is based on a refinement of an integral duality method introduced by L.C. Evans, and exploits stability properties of continuity and transport equations with rough velocity fields.

Dirichlet problems involving the 1-Laplacian and general nonlinearities

Francescantonio Oliva
Sapienza Università di Roma

June 9, 12:15

We give a brief overview of Dirichlet problems associated to

$$-\Delta_1 u = F(x, u, Du) \qquad \text{in } \Omega, \tag{1}$$

where $\Delta_1 u = \operatorname{div}(Du|Du|^{-1})$ is the 1-Laplacian operator and Ω is an open, bounded subset of $\mathbb{R}^N$ with regular boundary.

We assume different hypotheses on F in order to deduce existence, uniqueness, and regularity of BV-solutions to (1).

A distributional approach to fractional Sobolev and BV spaces

Giorgio Stefani
Scuola Internazionale Superiore di Studi Avanzati

June 9, 09:30

For $p \in [1, \infty]$, we introduce the distributional fractional Sobolev $S^{\alpha,p}$ and $BV^{\alpha,p}$ spaces exploiting the notion of fractional Riesz gradient of order $\alpha \in (0, 1)$. For $p \in (1, \infty)$, the space $S^{\alpha,p}$ coincides with the fractional Bessel potential space. The space $BV^{\alpha,p}$, instead, leads to a new notion of

distributional fractional perimeter, which generalizes the $W^{\alpha,1}$-perimeter. Our notion of distributional fractional perimeter allows to define a fractional analogue of De Giorgi's reduced boundary. Our main result extends De Giorgi's Blow-up Theorem to sets with locally finite distributional fractional perimeter, giving existence of blow-ups and a characterization of these (possibly non-unique) limit sets. We also discuss some recent developments of the distributional approach concerning fractional divergence-measure fields and fractional Leibniz rules.

On the sharp Hardy inequality in fractional Sobolev spaces

Anna Chiara Zagati
Università degli Studi di Parma

🕒 June 8, 14:45

We introduce the Hardy inequality on fractional Sobolev spaces $W^{s,p}(\Omega)$ and we look for the sharp constant on $\Omega \subset \mathbb{R}^N$ convex open set. Under the restriction $sp \geq 1$, our main tool is to construct supersolutions for the associated equation. Moreover, we observe that this method does not work for $sp < 1$, but we are able to overcome this limitation in the case $p = 2$. The results presented in this talk are obtained in collaboration with Francesca Bianchi (University of Parma), Lorenzo Brasco (University of Ferrara) and Firoj Sk (Okinawa Institute of Science and Technology).

Venue

All talks will take place at the Mathematics Department of the Alma Mater Studiorum Università di Bologna, located in Piazza di Porta S. Donato 5, Bologna.

The room is **Aula Vitali**, on the ground floor.

Social dinner

The social dinner will be held on Thursday June 8 at

Ristorante Lambrusco - Crescentine e Tigelle

located in via Ferruccio Garavaglia 5b, Bologna. It is at walking distance from the Mathematics Department, outside the city centre.

We meet at **20:00** directly in front of the restaurant.

We regret that, for organizational and financial reasons, the dinner is organized and reimbursed for **speakers only**. If someone wishes to participate, please write at

`berardo.ruffini@unibo.it`

before May 26th. In the message, please notify if there is any specific request (allergy, diet,...).

Programme

	Thursday 8 June	**Friday 9 June**
09:30 – 10:15		Stefani
10:15 – 11:00		Bianchi
11:00 – 11:30		*break*
11:30 – 12:15		De Luca
12:15 – 13:00		Oliva
13:00 – 14:00	*welcome*	
14:00 – 15:45	Ciani	
14:45 – 15:30	Zagati	
15:30 – 16:00	*break*	
16:00 – 16:45	Goffi	
16:45 – 17:30	Giovannardi	
20:00	*Dinner*	