L^p contraction estimates for parabolic equations via the nonlinear adjoint method

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In this talk I will report on some works in progress concerning stability and contraction estimates in Lebesgue spaces, along with uniqueness properties, for linear and nonlinear time-dependent PDEs with local and/or nonlocal diffusion. I will first discuss a general strategy to obtain L^1 contraction estimates for nonlinear diffusion equations of generalized porous medium type. Then, I will outline some possible applications to other advection and convection-diffusion models, together with Kruzkov type L^1 stability estimates for solutions of conservation laws. Finally, I will focus on contraction and stability estimates in L^p spaces for first- and second-order Hamilton-Jacobi equations along with the rate of convergence of the vanishing viscosity process. The latter results improve (with respect to the norm involved and the dependence on the constants) upon a rate of convergence obtained by M.G. Crandall, P.-L. Lions, and P.-L. Lions through viscosity solutions and probabilistic methods, and a more recent one by C.-T. Lin and E. Tadmor in L^1. This analysis is based on a refinement of an integral duality method introduced by L.C. Evans, and exploits stability properties of continuity and transport equations with rough velocity fields.