A distributional approach to fractional Sobolev and BV spaces

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For $p \in [1,\infty]$, we introduce the distributional fractional Sobolev $S^{\alpha,p}$ and $BV^{\alpha,p}$ spaces exploiting the notion of fractional Riesz gradient of order $\alpha \in (0,1)$. For $p \in (1,\infty)$, the space $S^{\alpha,p}$ coincides with the fractional Bessel potential space. The space $BV^{\alpha,p}$, instead, leads to a new notion of distributional fractional perimeter, which generalizes the $W^{\alpha,1}$-perimeter. Our notion of distributional fractional perimeter allows to define a fractional analogue of De Giorgi's reduced boundary. Our main result extends De Giorgi's Blow-up Theorem to sets with locally finite distributional fractional perimeter, giving existence of blow-ups and a characterization of these (possibly non-unique) limit sets. We also discuss some recent developments of the distributional approach concerning fractional divergence-measure fields and fractional Leibniz rules.